Physics for Computer Science Students
Lecture 7

MECHANICS OF CONTINUOUS MEDIA

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Introduction

Today: mechanics of continuous media!

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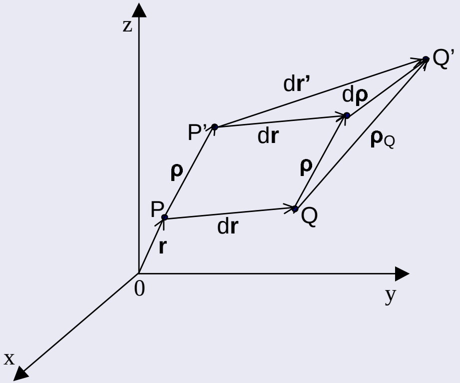
Displacement

Displacement vector



Rys. 1: Positions of two points before and after the displacement

Displacement vector

Notation

$\mathbf{r}(x, y, z)$ – position vector of the point P;
$d\mathbf{r} = \overrightarrow{PQ} = (dx, dy, dz)$ – relative position vector of the point Q with respect to the position of the point Q;
$\mathbf{r} + d\mathbf{r} = (x + dx, y + dy, z + dz)$ – position vector of the point Q with respect to the origin of the co-ordinate system;
$\boldsymbol{\rho} = \overrightarrow{PP'} = (\xi, \eta, \zeta)$ – displacement vector of the point P;
$\boldsymbol{\rho}_Q = \overrightarrow{QQ'} = \boldsymbol{\rho} + d\boldsymbol{\rho}$ – displacement vector of the point Q;
$d\mathbf{r}' = \overrightarrow{P'Q'} = d\mathbf{r} + d\boldsymbol{\rho}$ – relative position vector of the point Q' with respect to the position of the point P'.

Displacement

It is seen from Fig. 1 that $|d\mathbf{r}| \neq |d\mathbf{r}'|$. This is **deformation** of the material medium.

Tensor of the relative displacement

$$\begin{aligned} d\xi &= \frac{\partial \xi}{\partial x} dx + \frac{\partial \xi}{\partial y} dy + \frac{\partial \xi}{\partial z} dz \,, \\ d\eta &= \frac{\partial \eta}{\partial x} dx + \frac{\partial \eta}{\partial y} dy + \frac{\partial \eta}{\partial z} dz \,, \\ d\zeta &= \frac{\partial \zeta}{\partial x} dx + \frac{\partial \zeta}{\partial y} dy + \frac{\partial \zeta}{\partial z} dz \,, \end{aligned} \tag{1}$$

$$d\rho = T \, d\mathbf{r} \,, \tag{2}$$

T – tensor of the relative displacement.

Displacement

It is seen from Fig. 1 that $|d\mathbf{r}| \neq |d\mathbf{r}'|$. This is **deformation** of the material medium.

Tensor of the relative displacement

$$
\begin{aligned}
d\xi &= \frac{\partial \xi}{\partial x} dx + \frac{\partial \xi}{\partial y} dy + \frac{\partial \xi}{\partial z} dz \,, \\
d\eta &= \frac{\partial \eta}{\partial x} dx + \frac{\partial \eta}{\partial y} dy + \frac{\partial \eta}{\partial z} dz \,, \\
d\zeta &= \frac{\partial \zeta}{\partial x} dx + \frac{\partial \zeta}{\partial y} dy + \frac{\partial \zeta}{\partial z} dz \,,
\end{aligned}
\tag{1}
$$

$$
d\boldsymbol{\rho} = T \, d\mathbf{r} \,, \tag{2}
$$

T – tensor of the relative displacement.

Tensor of the relative displacement

It is seen from Fig. 1 that

$$d\mathbf{r}' = d\mathbf{r} + d\boldsymbol{\rho}\,, \tag{3}$$

$$d\mathbf{r}' = d\mathbf{r} + T\,d\mathbf{r} = (1+T)d\mathbf{r}\,, \tag{4}$$

"1" – unit tensor

$$\|\delta_{\mu\nu}\| = \left\| \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right\| . \tag{5}$$

Notation: $T' = 1 + T$, and

$$d\mathbf{r}' = T' d\mathbf{r}\,. \tag{6}$$

Displacement

Tensor of the relative displacement T – in general is not a symmetric tensor. Let us decompose it on the symmetric $T^{(s)}$ and antisymmetric $T^{(a)}$ parts:

$$T = T^{(s)} + T^{(a)}, \tag{7}$$

$$T^{(s)} = \left\| \begin{array}{ccc} T_{xx} & \frac{1}{2}(T_{xy} + T_{yz}) & \frac{1}{2}(T_{xz} + T_{zx}) \\ \frac{1}{2}(T_{xy} + T_{yz}) & T_{yy} & \frac{1}{2}(T_{yz} + T_{zy}) \\ \frac{1}{2}(T_{xz} + T_{zx}) & \frac{1}{2}(T_{xz} + T_{zx}) & T_{zz} \end{array} \right\| \tag{8}$$

$$T^{(a)} = \left\| \begin{array}{ccc} 0 & \frac{1}{2}(T_{xy} - T_{yz}) & \frac{1}{2}(T_{xz} - T_{zx}) \\ -\frac{1}{2}(T_{xy} - T_{yz}) & 0 & \frac{1}{2}(T_{yz} - T_{zy}) \\ -\frac{1}{2}(T_{xz} - T_{zx}) & \frac{1}{2}(T_{yz} - T_{zy}) & 0 \end{array} \right\| \tag{9}$$

Displacement

Let us introduce the vector

$$\mathbf{T}^{(a)} = \mathbf{i}\,\frac{1}{2}(T_{zy} - T_{yz}) + \mathbf{j}\,\frac{1}{2}(T_{xz} - T_{zx}) + \mathbf{k}\,\frac{1}{2}(T_{yz} - T_{xy}) \quad (10)$$

Making use of the definition of the tensor T (compare (1) and (2))

$$2\mathbf{T}^{(a)} = \mathbf{i}\left(\frac{\partial\zeta}{\partial y} - \frac{\partial\eta}{\partial z}\right) + \mathbf{j}\left(\frac{\partial\xi}{\partial z} - \frac{\partial\zeta}{\partial x}\right) + \mathbf{k}\left(\frac{\partial\eta}{\partial x} - \frac{\partial\xi}{\partial y}\right) \quad (11)$$

Notation: $\mathbf{T}^{(a)} = \mathbf{u}$

$$\mathbf{u} = \frac{1}{2}\,\mathrm{rot}\,\boldsymbol{\rho} \quad (12)$$

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Deformation

Notation: $\mathbf{T}^{(s)} = \mathbf{T}^{(d)}$ – tensor of the pure deformation (d like deformation)

$$\mathbf{T}^{(d)} = \left\| \begin{array}{ccc} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{array} \right\| = \left\| \begin{array}{ccc} \varepsilon_{x} & \gamma_{z} & \gamma_{y} \\ \gamma_{z} & \varepsilon_{y} & \gamma_{x} \\ \gamma_{y} & \gamma_{x} & \varepsilon_{z} \end{array} \right\| \tag{13}$$

$$\begin{aligned} \varepsilon_x &= \frac{\partial \xi}{\partial x}, \quad \gamma_x = \frac{1}{2}\left(\frac{\partial \eta}{\partial z} + \frac{\partial \zeta}{\partial y}\right) \\ \varepsilon_y &= \frac{\partial \eta}{\partial y}, \quad \gamma_y = \frac{1}{2}\left(\frac{\partial \zeta}{\partial x} + \frac{\partial \xi}{\partial z}\right) \\ \varepsilon_z &= \frac{\partial \zeta}{\partial z}, \quad \gamma_z = \frac{1}{2}\left(\frac{\partial \xi}{\partial y} + \frac{\partial \eta}{\partial x}\right) \end{aligned} \tag{14}$$

Deformation

ε_x, ε_z, ε_z – longitudinal deformation
γ_x, γ_y, γ_z – transversal deformation

It can be easily shown that for an arbitrary vector $\mathbf{a}$ and antisymmetric tensor T

$$T^{(a)}\,\mathbf{a} = \mathbf{T}^{(a)} \times \mathbf{a} \qquad (15)$$

where vector $\mathbf{T}^{(a)}$ has a form (10)

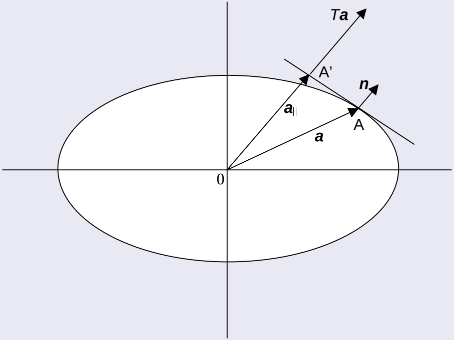
$$\mathbf{T}^{(a)} = \mathbf{i}\,\frac{1}{2}(T_{zy} - T_{yz}) + \mathbf{j}\,\frac{1}{2}(T_{xz} - T_{zx}) + \mathbf{k}\,\frac{1}{2}(T_{yz} - T_{xy})$$

i.e.

$$d\boldsymbol{\rho} = T^{(d)}\,d\mathbf{r} + \mathbf{u} \times d\mathbf{r}\,. \qquad (16)$$

Geometrical interpretation of the symmetric tensor

Every symmetric tensor can be brought to the main axes



Rys. 2: Geometrical construction of the vector $T\mathbf{a}$ with the help of the tensorial quadric

Geometrical interpretation of the symmetric tensor

Quadric equation

Let us consider all vectors **a** satisfying the equation

$$\mathbf{a}\, T\, \mathbf{a} = F(a_x, a_y, a_z) = \text{const} \neq 0\,. \tag{17}$$

$$F = T_{xx} a_x^2 + T_{yy} a_y^2 + T_{zz} a_z^2 + 2T_{xy} a_x a_y + 2T_{yz} a_y a_z + 2T_{zx} a_z a_x\,. \tag{18}$$

This is an equation of the surface of the second order with the center at the beginning of the vector **a** – **tensorial quadric** – geometrical representation 9 of the symmetric tensor T.

$$T\mathbf{a} = \frac{1}{2}(\mathbf{i}\frac{\partial F}{\partial a_x} + \mathbf{j}\frac{\partial F}{\partial a_y} + \mathbf{k}\frac{\partial F}{\partial a_z}) = \frac{1}{2}\,\mathrm{grad}\, F\,, \tag{19}$$

i.e. the vector $T\mathbf{a}$ is parallel to the normal vector **n**.

Geometrical interpretation of the symmetric tensor

In general vectors **a** and $T\mathbf{a}$ have different directions and as it is seen from Fig. 2. Both vectors are parallel when the vector **a** lies on the one of the three main axes of the tensorial quadric. In the rectangular co-ordinate system u, v, w with axes along the main quadric axes and with versors $\mathbf{i}_u, \mathbf{j}_v, \mathbf{k}_w$, one has

$$\mathbf{a}\,T\mathbf{a} = T_{uu}a_u^2 + T_{vv}a_v^2 + T_{ww}a_w^2\,. \tag{20}$$

Vector $T\mathbf{a}$

$$T\mathbf{a} = \mathbf{i}_u\; T_{uu}a_u + \mathbf{j}_v\; T_{vv}a_v + \mathbf{k}_w\; T_{ww}a_w\,, \tag{21}$$

has components on the main axes elongated with respect to the vector **a** $\{T_{uu}, T_{vv}, T_{ww}\}$ -times . This is the origin of the word **tensor**, od (*lat. tendo, tentendi, tentum*) or more poetic *tensum* – **elongate**.

Main elongations

$$\mathbf{T}^{(d)} = \left\| \begin{matrix} \varepsilon_u & 0 & 0 \\ 0 & \varepsilon_v & 0 \\ 0 & 0 & \varepsilon_w \end{matrix} \right\| . \tag{22}$$

$\varepsilon_u, \varepsilon_v, \varepsilon_w$ – main elongations.

$$d\mathbf{r} = \mathbf{i}_u du + \mathbf{j}_v dv + \mathbf{k}_w dw . \tag{23}$$

From (22) and (23) $\rightsquigarrow$

$$d\boldsymbol{\rho}_d = T^{(d)} d\mathbf{r} = \mathbf{i}_u \varepsilon_u du + \mathbf{j}_v \varepsilon_v dv + \mathbf{k}_w \varepsilon_w dw . \tag{24}$$

Quadric of the tensor T

$$d\mathbf{r} T^{(d)} d\mathbf{r} = \varepsilon_u du^2 + \varepsilon_v dv^2 + \varepsilon_w dw^2 . \tag{25}$$

Main elongations

Interpretation of the main elongations

From (24) $\rightsquigarrow$

$$d\xi_d = \varepsilon_u du,\ d\eta_d = \varepsilon_v dv,\ d\zeta_d = \varepsilon_w dw\,, \tag{26}$$

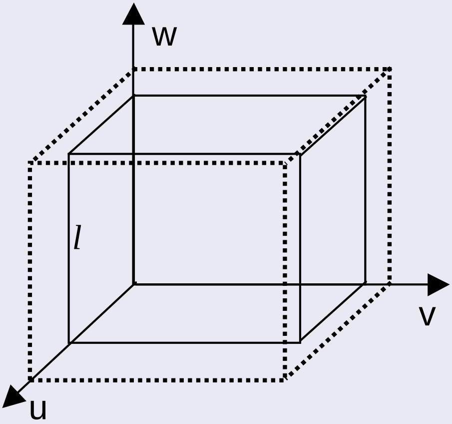
i.e.

$$\varepsilon_u = \frac{d\xi_d}{du},\ \varepsilon_v = \frac{d\eta_d}{dv},\ \varepsilon_w = \frac{d\zeta_d}{dw}\,. \tag{27}$$

Main elongation ε_u means the relative change of the distance, i.e. the change of the distance on the unit of length.
If before the deformation the distance between two points was du, then after the displacement it was

$$du + d\xi_d = (1+\varepsilon_u)du\,. \tag{28}$$

The proper volume strain



Rys. 3: Change of the volume of the cube caused by the deformation

The proper volume strain

The volume of a cube

$$V = l^3 \,. \tag{29}$$

Caused by deformation the cube edges are elongated:

$$\Delta l_u = l(1+\varepsilon_u),\ \Delta l_v = l(1+\varepsilon_v),\ \Delta l_w = l(1+\varepsilon_w)\,. \tag{30}$$

The new cube volume:

$$V' = l^3(1+\varepsilon_u)(1+\varepsilon_v)(1+\varepsilon_w)\,. \tag{31}$$

ε_i – is very small, so $V' = l^3(1+\varepsilon_u+\varepsilon_v+\varepsilon_w)$. The change of the volume:

$$\Delta V = V' - V \,. \tag{32}$$

The relative change of the volume (on the unit of volume):

$$\frac{\Delta V}{V} = \varepsilon_u + \varepsilon_v + \varepsilon_w \,. \tag{33}$$

The proper volume strain

The sum of the components on the main diagonal of the tensor is an invariant with respect to the the change of the co-ordinate system (the trace), so

$$\frac{\Delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z \,, \tag{34}$$

but

$$\frac{\Delta V}{V} = \frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} + \frac{\partial \zeta}{\partial z} \,, \tag{35}$$

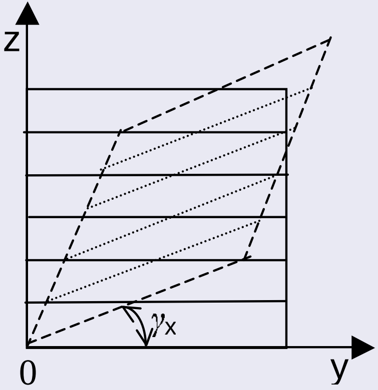
i.e. **the proper volume strain** θ

$$\theta = \frac{\Delta V}{V} = \mathrm{div}\, \boldsymbol{\rho} \,, \tag{36}$$

where

$$\theta = \varepsilon_x + \varepsilon_y + \varepsilon_z \,. \tag{37}$$

Transversal deformation



Rys. 4: Shearing of the cube in the plane y, z

Transversal deformation

From the definition of the tensor $T^{(d)}$ (Eqn (13))

$$\begin{aligned} d\xi_d &= \varepsilon_x dx + \gamma_z dy + \gamma_y dz \, , \\ d\eta_d &= \gamma_z dx + \varepsilon_y dy + \gamma_x dz \, , \\ d\zeta_d &= \gamma_y dx + \gamma_x dy + \varepsilon_z dz \, . \end{aligned} \tag{38}$$

Let us assume that only $\gamma_x \neq 0$, the rest vanishes. In such a case:

$$d\xi_d = 0 \, , \; d\eta_d = \gamma_x dz \, , \; d\zeta_d = \gamma_x dy \, . \tag{39}$$

Transversal deformation

- Points on the axis x: $\rightsquigarrow$ $dy = dz = 0$ – they do not change the positions;
- Points on the axis y: $\rightsquigarrow$ $dx = dz = 0$ – there is a translation in the direction of the axis z proportional to dy, and axis y rotates in the direction of axis z by the angle γ_x ($\operatorname{tg}\gamma_x \approx \gamma_x$);
- Points on the axis z: $\rightsquigarrow$ $dx = dy = 0$ – rotation of the axis z in the direction of the axis y by the angle γ_x.

In particular the square on the plane perpendicular to the axis x, take the form of a rhombus (compare Fig. 4) it is a change of the shape without changing a volume.

Kinematics of the continuous media

Definition of the velocity

Velocity: it is a vector

$$\mathbf{v}(x,y,z,t) = \frac{\partial \boldsymbol{\rho}(x,y,z,t)}{\partial t} = \left(\frac{\partial \xi}{\partial t}, \frac{\partial \eta}{\partial t}, \frac{\partial \zeta}{\partial t}\right)(x,y,z,t)\,. \qquad (40)$$

Definition of the acceleration

Acceleration: it is a vector

$$\mathbf{a} = (\mathbf{v}\,\mathrm{grad})\mathbf{v} + \frac{\partial \mathbf{v}}{\partial t}\,. \qquad (41)$$

Local and substantial derivatives

Let us consider a certain physical quantity φ – scalar, vector or tensor:

$$\varphi = \varphi(\mathbf{r}, t) = \varphi(x, y, z, t) \, .$$

One can proceed in two ways:

1. observe the changing of the φ in the define3d point of the space przestrzeni;
2. observe the changing of the φ for the defined and traveling the point of the medium.

Local and substantial derivatives

Local derivative

Ad 1. Change of φ in the defined point of a space $\mathbf{r}$ defines the local derivative of the quantity φ.

$$\frac{\partial\varphi}{\partial t} = \lim_{\Delta t\to 0} \frac{\varphi(\mathbf{r}, t+\Delta t) - \varphi(\mathbf{r}, t)}{\Delta t}. \tag{42}$$

Substantial derivative

Ad 2. $\varphi(\mathbf{r}, t)$ – value in the instant t at the point $\mathbf{r}$.

$$\frac{d\varphi}{dt} = \lim_{\Delta t\to 0} \frac{\varphi(\mathbf{r}+\mathbf{v}\Delta t, t+\Delta t) - \varphi(\mathbf{r}, t)}{\Delta t}. \tag{43}$$

Local and substantial derivatives

Substantial derivative

After developing into series and neglecting the higher order terms

$$\frac{d\varphi}{dt} = (\mathbf{v}\,\mathrm{grad})\varphi + \frac{\partial\varphi}{\partial t}\,. \tag{44}$$

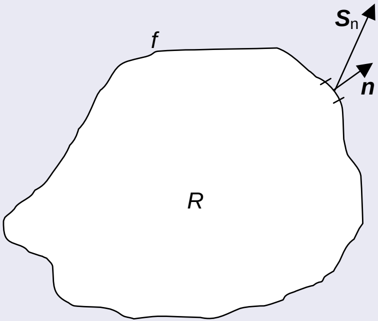
Conclusions

1. Velocity $\mathbf{v}$ is the local derivative of the displacement vector with respect to time (compare Eqn (40)):
$$\mathbf{v} = \frac{\partial\boldsymbol{\rho}}{\partial t}\,.$$

2. Acceleration a is the substantial derivative of the velocity vector $v(r,t)$ with respect to time (compare Eqn (41)):
$$\mathbf{a} = \frac{d\varphi}{dt} = (\mathbf{v}\,\mathrm{grad})\mathbf{v} + \frac{\partial\mathbf{v}}{\partial t}\,.$$

Dynamics of the continuous media

Tension vector



Rys. 5: Tension vector

Dynamics of the continuous media

Tension vector

Tension vector $\mathbf{S}_n df$ – describes the interaction of two parts of the continuous media divided by the imaginary arbitrary surface; it is a surface force with which the element df, pointed by the normal vector $\mathbf{n}$, acts on the opposite part of the body.

Dimension of the tension: $\frac{[\text{siła}]}{[\text{cm}]^2}$

Dimension of the force: ????????

Dynamics of the continuous media

Gauss theorem

$$\int_R \mathrm{div}\ T dv = \int_S T\,\mathbf{n} df\ . \tag{45}$$

Tension vector

Tension vector can be represented by the tensor:

$$\mathbf{S}_n = S\,\mathbf{n}\,. \tag{46}$$

gdzie

$$S = \left\| \begin{array}{ccc} S_{xx} & S_{yx} & S_{zx} \\ S_{xy} & S_{yy} & S_{zy} \\ S_{xz} & S_{yz} & S_{zz} \end{array} \right\| \tag{47}$$

Dynamics of the continuous media

$$\int_F \mathbf{S}_n df = \int_F S\mathbf{n} df = \int_R \operatorname{div} S d\tau \,. \tag{48}$$

Equation of motion

$$\int_R \rho_m \frac{d\mathbf{v}}{dt} = \int_R \rho_m \mathbf{F} d\tau + \int_F \mathbf{S}_n df \,, \tag{49}$$

ρ_m – mass density of the medium; $\mathbf{F}$ – external force acting on the mass unit; $\mathbf{S}_n$ – surface tension.

$$\int_R \left(\rho_m \frac{d\mathbf{v}}{dt} - \rho_m \mathbf{F} - \operatorname{div} S \right) d\tau = 0 \,. \tag{50}$$

Dynamics of the continuous media

Equation of motion

$$\rho_m \frac{d\mathbf{v}}{dt} = \rho_m \mathbf{F} + \operatorname{div} S\,. \qquad (51)$$

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Elastic medium

Ideally elastic medium

tensions $S_{\mu\nu}$ are the unique functions of strains ε_{mn}:

$$S_{\mu\nu} = f_{\mu\nu}(\varepsilon_{mn})\,. \tag{52}$$

It can be shown that the tension tensor $S_{\mu\nu}$ is symmetric

$$S = \left\| \begin{array}{ccc} \sigma_x & \tau_z & \tau_y \\ \tau_z & \sigma_y & \tau_x \\ \tau_y & \tau_x & \sigma_z \end{array} \right\| . \tag{53}$$

Elastic medium

Hooke's law

Robert Hooke declared his law in 1676 in the form of an anagram:

ceiiinosssttvu

what means **ut tensio sic vis**
what means **a force is so big as stretching**

Elastic medium

Ideally elastic medium

Develop Eqn (54) into series, omit the higher order terms in order to obtain the generalized equation of motion: components of the tension tensor are the linear functions of the strain tensor components at the every point of the elastic body: $\sigma_{ij} = c_{ijkl}\varepsilon_{kl}$. Eg. (when no initial tensions then the constant quantities vanish):

$$\sigma_x = c_{11}\varepsilon_x + c_{12}\varepsilon_y + c_{13}\varepsilon_z + c_{14}2\gamma_x + c_{15}2\gamma_y + c_{16}2\gamma_z \qquad (54)$$

$$\tau_x = c_{41}\varepsilon_x + c_{42}\varepsilon_y + c_{43}\varepsilon_z + c_{44}2\gamma_x + c_{45}2\gamma_y + c_{46}2\gamma_z \qquad (55)$$

Elastic medium

Energy

During the deformation external forces volume forces and the surface forces execute a certain work, which partially is changed into the kinetic and partially is changed into the potential energy. We have

$$\delta U + \delta E_k = \delta A + \delta Q\,, \tag{56}$$

δU – increase of the potential energy; δE_k – increase of the kinetic energy; δQ – supplied heat; $\delta A = \delta A_S + \delta A_p$: A_p – work executed by the mass forces, A_S – work executed by the surface forces.

When heat is not supplied then δV is the total differential (conclusion from thermodynamics)

Elastic medium

Lowering the number of the elastic constants

In general the number of the elastic constants equals 81. When no initial tensions and $c_{\mu\nu} = c_{\nu\mu}$, the number of the elastic constants is reduced to 21.

Isotropic body

elastic potential does not depend on the change of the change of the co-ordinate system, i.e. it can be expressed with the help of the invariants.

Elastic medium

Isotropic body – invariants

$$J_1 = \varepsilon_x + \varepsilon_y + \varepsilon_z \,, \tag{57}$$

$$J_2 = \begin{vmatrix} \varepsilon_x & \gamma_z \\ \gamma_z & \varepsilon_y \end{vmatrix} + \begin{vmatrix} \varepsilon_y & \gamma_x \\ \gamma_x & \varepsilon_z \end{vmatrix} + \begin{vmatrix} \varepsilon_z & \gamma_y \\ \gamma_y & \varepsilon_z \end{vmatrix} \tag{58}$$

$$J_3 = \begin{vmatrix} \varepsilon_x & \gamma_z & \gamma_y \\ \gamma_z & \varepsilon_y & \gamma_y \\ \gamma_y & \gamma_x & \varepsilon_z \end{vmatrix} \tag{59}$$

Elastic medium

Isotropic body – invariants

Isotropic elastic body without initial tensions

$$v(J_1, J_2) = AJ_1^2 + BJ_2 > 0 \tag{60}$$

(two elastic constants only, J_3 is absent, because it is the quantity of the third order).

Isotropic elastic body with initial tensions

$$v(J_1, J_2) = -PJ_1 + AJ_1^2 + BJ_2 > 0 \tag{61}$$

$A = 0,\ B = 0$ tensions create spherically-symmetric tension, identical in all directions. Such situation occurs in liquids:

$$\begin{aligned} \sigma_x = \sigma_y = \sigma_z = -P\,, \\ \tau_x = \tau_y = \tau_z = 0\,. \end{aligned} \tag{62}$$

:-)

The end of the lecture 7