Physics for Computer Science Students
Lecture 5

GRAVITATION

Romuald Kotowski

Department of Applied Informatics

PJIIT 2009

Spis treści

Spis treści

Spis treści

Spis treści

Content

Sky pictures: 1

Fig. 1: Cloud

Sky pictures: 2

Fig. 2: Sombrero

Sky pictures: 3

Fig. 3: Whirlpool

Sky pictures: 4

Fig. 4: Helix-nebula

Sky pictures: 5

Fig. 5: Planets as seen from the Luna

Sky pictures: 6

Fig. 6: The Earthrise seen from the Luna

Content

Astronomy

Astronomy – the oldest science

$$[gr.\ astron = the\ star + n\acute{o}mos = the\ law]$$

The astronomy started as the applied science - time measurements and establishing the calendar were the first jobs. Some facts indicate that the first Luna observations were made 30 000 years BC. The length of the year was known at the beginning of the third milenium BC in the ancient Egypt in the era of the pyramids construction. At the same time in Europe the megalith space observatories were builded:

- Stonehenge (Wales)
- Carnac (France)

It was so called horizontal astronomy:

- places of Sun, Luna and other 'stars' rises and sets
- the shortest and the longest days

Merhirs in Stonehenge

Fig. 7: Megaliths in Stonehenge, Wales

Merhirs in Carnac

Fig. 8: The stone statues in Carnac, Bretony - France

The greatest in the world center of the megaliths. About 4000-5000 merhirs on the area of 4km^2 to the north of Carnac. It is older than Stonehenge or Egypt pyramids.

Many of them have found new applications as crosses and the saint's monuments. Nevertheless there is still something to see. . .

Pythagoras

Pythagoras of Samos (about 569 BC - about 475 BC) – a Greek philosopher who made important developments in mathematics, astronomy, and the theory of music. The theorem now known as Pythagoras's theorem was known to the Babylonians 1000 years earlier but he may have been the first to prove it.
Pythagoras founded a philosophical and religious school in Croton (now Crotone, on the east of the heel of southern Italy) that had many followers. Pythagoras was the head of the society with an inner circle of followers known as mathematikoi. The mathematikoi lived permanently with the Society, had no personal possessions and were vegetarians. They were taught by Pythagoras himself and obeyed strict rules.

Pythagoras

The beliefs that Pythagoras held were:

1. that at its deepest level, reality is mathematical in nature,
2. that philosophy can be used for spiritual purification,
3. that the soul can rise to union with the divine,
4. that certain symbols have a mystical significance, and
5. that all brothers of the order should observe strict loyalty and secrecy.

Pythagoras believed that all relations could be reduced to number relations – the quantity and proportions are important.

Pythagorean tuning – is a system of musical tuning in which the frequency relationships of all intervals are based on the ratio 3:2. Its name comes from medieval texts which attribute its discovery to Pythagoras, but its use has been documented as long ago as 3500 B.C. in Babylonian texts. It is the oldest way of tuning the 12-note chromatic scale.

Phytagorean

Phytagorean considered mathematics as divided into four parts:

1. geometry,
2. arithmetics,
3. astronomy,
4. music.

Astronomy – applied geometry, music – applied arithmetics.
These four parts of science were called **quadrivium** [*Latin, four ways*] fundamental teaching at the medieval universities.

New terminology introduced by Phytagorean: philosophy and cosmos, [*gr. kósmos = order, in contradiction to chaos*].

New planet

September 2006 – american astronomers informed about the discovery of the very big and very bright planet revolved around the very distant star – it may lead to the revision of the theory on the establishing of planets.

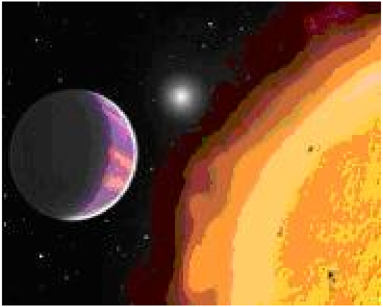

Fig. 9: New planet HAT-P-1

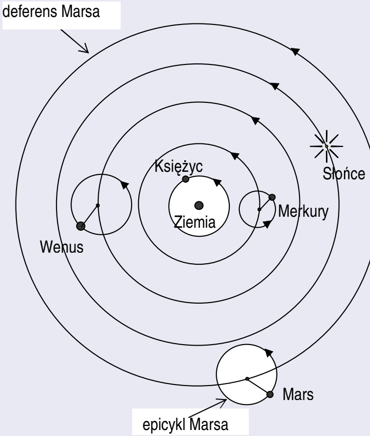

Fig. 10: Solar system of Claúdios Ptolemeus (*Almagest*, ∼150)

It is worth of mention that the epicycles in the Copernicus theory still exist, but the number of them is significantly reduced. ich liczbę.

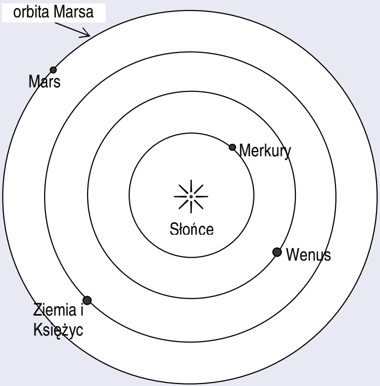

Fig. 11: Solar system of Mikołaj Kopernik (*De revolutionibus orbium coelestium*, 1543)

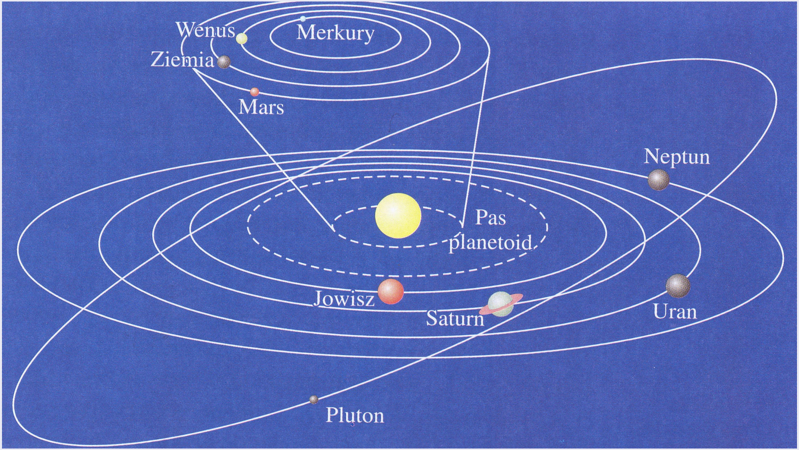

Fig. 12: Solar system

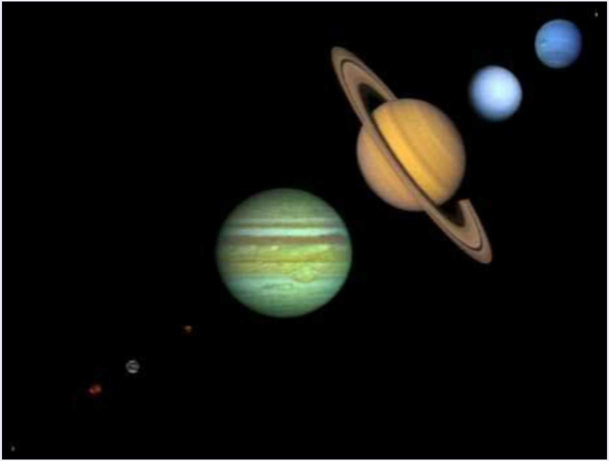

Fig. 13: The eight planets and Pluto with approximately correct relative sizes

The Big Questions

- What is the origin of the solar system? It is generally agreed that it condensed from a nebula of dust and gas. But the details are far from clear.
- How common are planetary systems around other stars? There is now good evidence of Jupiter-sized objects orbiting several nearby stars. What conditions allow the formation of terrestrial planets? It seems unlikely that the Earth is totally unique but we still have no direct evidence one way or the other.
- Is there life elsewhere in the solar system? If not, why is Earth special?
- Is there life beyond the solar system? Intelligent life?
- Is life a rare and unusual or even unique event in the evolution of the universe or is it adaptable, widespread and common?

Answers to these questions, even partial ones, would be of enormous value.

Astronomical units

Name	Magnitude	Earth-Sun
Astr. year [AU]	$1\ \mathrm{AU} \approx 1.496 \cdot 10^{11}\ \mathrm{m}$	1
Parsec [pc]	$1\ \mathrm{pc} \approx 3.086 \cdot 10^{16}\ \mathrm{m}$ $\approx 2.063 \cdot 10^{5}\ \mathrm{AU}$	0.000005
Light year [ly]	$1\ \mathrm{ly} \approx 9.461 \cdot 10^{15}\ \mathrm{m}$ $\approx 6.324 \cdot 10^{4}\ \mathrm{AU}$ $\approx 0.3066\ \mathrm{pc}$	500 s

Distances between the Planets and the Sun

Object	AU	ly	mln km
Mercury	0.38	3 min 10 s	57.9
Venus	0.72	6 min	108.2
Earth	1	8 min 20 s	149.6
Mars	1.52	12 min 40 s	227.9
Jupiter	5.2	43 min 20 s	778.6
Saturn	9.54	1 h 19 min 30 s	1 433.5
Uranus	19.19	2 h 39 min 55 s	2 872.5
Neptune	30.6	4 h 10 min 30 s	4 495.1
Pluto	39.53	5 h 29 min 25 s	5 906.4
Planetoids	2 - 3		
Pas Kuipera	$\sim$ 30 - 50		

Kepler's laws of planetary motion

Izaak Newton (1643 - 1727) has formulated his **law of universal gravitation** on the results of the astronomical observations of the motions of planets. These observations helped Johannes Kepler (1571-1630) to formulate the three laws:

1. The orbit of every planet is an ellipse with the sun at a focus
2. A line joining a planet and the sun sweeps out equal areas during equal intervals of time.
3. The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.

Ellipse

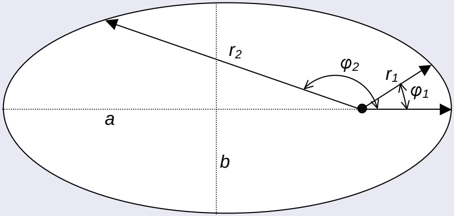

Fig. 14: Geometry of ellipse

$$p = \frac{b^2}{a}, \qquad \varepsilon = \sqrt{1 - \frac{b^2}{a^2}} \tag{1}$$

p – parametr; ε – eccentricity; a, b – half-axes of ellipse.

Ellipse

the equation of ellipse in the polar co-ordinates

$$r = \frac{p}{1 + \varepsilon \cos\varphi}\,, \tag{2}$$

After differentiation

$$\frac{1}{p} = \frac{d^2}{d\varphi^2}\left(\frac{1}{r}\right) + \frac{1}{r}\,. \tag{3}$$

Remark: Acceleration in the polar co-ordinates:

$$a_r = \ddot{r} - r\dot{\varphi}^2\,, \qquad a_\varphi = 2\dot{r}\,\dot{\varphi} + r\,\ddot{\varphi}\,. \tag{4}$$

The field velocity

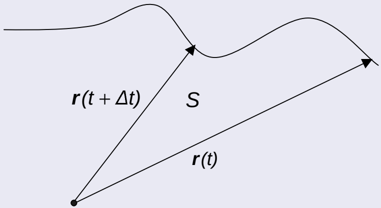

Fig. 15: The field velocity

The field velocity vector is perpendicular to the surface of the motion and direction according to the right-handed screw rule and the length equal numerically to area of the field S circled by the indicating vector $\mathbf{r}$ during the time t..

$$\mathbf{c} = \frac{1}{2}\mathbf{r} \times \mathbf{v}\,. \tag{5}$$

The field velocity

It follows

$$c = |\mathbf{c}| = \frac{1}{2}|\mathbf{r} \times \mathbf{v}| = \frac{1}{2} r\, v_\varphi = r^2 \dot{\varphi}\,. \tag{6}$$

After differentiation

$$0 = r(2\dot{r}\,\dot{\varphi} + r\,\ddot{\varphi}) = r\, a_\varphi \tag{7}$$

the transversal component of the acceleration vanishes.
Conclusion: acceleration is always directed to the Sun, so the force has to be the central force!

The field velocity

Moreover:

$$\dot{r} = \dot{\varphi}\frac{dr}{d\varphi} = \frac{2c}{r^2}\frac{dr}{d\varphi} = -2c\frac{d}{d\varphi}\left(\frac{1}{r}\right), \tag{8}$$

czyli

$$\ddot{r} = \dot{\varphi}\frac{d}{d\varphi}(\dot{r}) = -\frac{4c^2}{r^2}\frac{d^2}{d\varphi^2}\left(\frac{1}{r}\right), \tag{9}$$

skąd

$$a_r = -\frac{4c^2}{p}\frac{1}{r^2} = k\,\frac{1}{r^2}, \tag{10}$$

so, the acceleration, **and the force** , are reciprocally proportional to the square of the distance from the Sun.

The third Kepler's law

Coefficient of r^{-2}

$$k = -\frac{4c^2}{p} = -\frac{4c^2 a}{b^2}\,, \tag{11}$$

The field velocity, iso

the area of the ellipse $\pi\, a\, b$ divided by the time of circulation T

$$k = -\frac{4a}{b^2}\left(\frac{\pi a b}{T}\right)^2 = -\frac{4\pi^2 a^3}{T^2}\,. \tag{12}$$

The quotient a^3/T^2 has the same value for all planets so the Sun attracts all planets according to the universal law: **there exists the one law of the universal gravity only**.

Newton's law of universal gravitation

Newton's law of universal gravitation is an empirical physical law describing the gravitational attraction between bodies with mass. It is a part of classical mechanics and was first formulated in Newton's work *Philosophiae Naturalis Principia Mathematica*, first published on July 5, 1687. In modern language it states the following:

Every point mass attracts every other point mass by a force pointing along the line intersecting both points. The force is proportional to the product of the two masses and inversely proportional to the square of the distance between the point masses:

$$F = G\frac{m_1\, m_2}{r^2}$$

where: F – the magnitude of the gravitational force between the two point masses, G – the gravitational constant, m_1 – the mass of the first point mass, m_2 – the mass of the second point mass, r – the distance between the two point masses.

Newton's law of universal gravitation

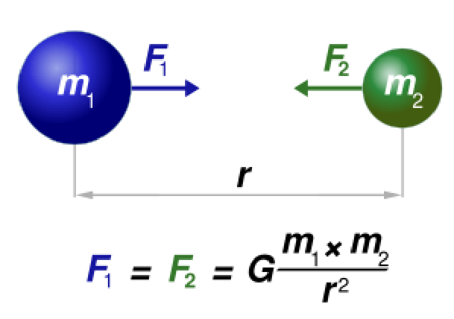

Fig. 16: The mechanisms of Newton's law of universal gravitation; a point mass m_1 attracts another point mass m_2 by a force F_2 which is proportional to the product of the two masses and inversely proportional to the square of the distance (r) between them. Regardless of masses or distance, the magnitudes of $|F_1|$ and $|F_2|$ will always be equal. G is the gravitational constant.

Newton's law of universal gravitation

Let us consider the body of the m_1 and the radius r_0 (eg. the Earth) and a small body of the mass m_2 and the small distance z over the Earth ($z \ll r_0$).

$$\mathbf{F}_{21} = G\frac{m_1 m_2}{r^2}\frac{\mathbf{r}_{12}}{r} \tag{13}$$

The universal constant of gravitation

$$G = (6.6732 \pm 0.0031) \cdot 10^{-11}\ \mathrm{m}^3 \cdot \mathrm{kg}^{-1} \cdot \mathrm{s}^{-2}\,.$$

The potential of the force (13) equals

$$V = -G\,\frac{m_1 m_2}{r}\,. \tag{14}$$

$r \longrightarrow r_0 + z$

$$V = -G\frac{m_1 m_2}{r_0(1+z/r_0)} \approx -\frac{Gm_1 m_2}{r_0} + \frac{Gm_1 m_2 z}{r_0^2}, \tag{15}$$

i.e.

$$V = m_2 g z + \text{const} \tag{16}$$

Earth acceleration

$$g = \frac{G\,m_1}{r_0^2}. \tag{17}$$

This is the approximation only, the influence of the rotation of the Earth should be taken into account (the centrifugal force) The mass of the Earth can be estimated from Eqn. (17).

Trajectories

The cone curves

Let us come back to Eqns (2) and (3)

$$(*) \qquad r = \frac{p}{1 + \varepsilon \cos \varphi} \,,$$

$$(**) \qquad \frac{1}{p} = \frac{d^2}{d\varphi^2}\left(\frac{1}{r}\right) + \frac{1}{r} \,.$$

Eqn. (*) is the equation of the cone curve with the beginning of the co-ordinate system in one of the focuses. The character of the curve is defined by the eccentricity ε.

Trajectories

The cone curves

Eccentricity	Effect
$\varepsilon < 1$	r – finite for all angles φ (**ellipse**)
$\varepsilon = 1$	r goes to the ∞ for $\cos\varphi = -1$, i.e. $\varphi = \pi$ (**parabola**)
$\varepsilon > 1$	from the condition $r > 0$, we have $\lvert\varphi\rvert < \arccos\varepsilon^{-1}$, so, the asymptotes (**hyperbola**)

Cosmic velocities

Cosmic velocities – the velocity to be reached by an arbitrary body (e.g. rocket, the space vehicle), in order its kinetic energy overcome the Earth gravitation and move away on the proper distance to be able to travel in the space without the additional propulsion.

There are several cosmic velocities. All of the were calculated by the polish engineer and astronomer Ary Sternfeld.

Cosmic velocities

The first cosmic velocity

The first cosmic velocity ($v_I = 7.91$ km/s) allows the object to orbit around the Earth or any other cosmic object.
This velocity can be calculated from the equations

$$a = \frac{F}{m} = \frac{GM}{R^2}\,, \tag{18}$$

and

$$a = \frac{v^2}{R}\,. \tag{19}$$

The first cosmic velocity

Stąd

$$v_I = \sqrt{\frac{GM}{R}}\,, \tag{20}$$

where G – gravitational constant, M – mass of the cosmic object, R – radius of the cosmic object.

For the Earth $v_I = 7.91$ km/s.

In reality the rockets starting to the East contain some additional velocity from the rotational motion of the Earth. On the equator this effect is the greatest: the benefit equals appr. 463 m/s.

The second cosmic velocity

The second cosmic velocity (escape velocity) - minimum velocity required for a spacecraft or other object to escape from the gravitational pull of a planetary body. In the case of the Earth, the escape velocity is 11.2 kps/6.9 mps; the Moon, 2.4 kps/1.5 mps; Mars, 5 kps/3.1 mps; and Jupiter, 59.6 kps/37 mps.
It means that the trajectory has to be the parabola or the hyperbola.
It can be calculated as a difference of the energy on the surface of a given cosmic object and in the infinity ($= 0$). On the surface it is the sum of the potential energy $E_p = -G\dfrac{Mm}{R}$ and the kinetic energy $E_k = \dfrac{mv^2}{2}$. It follows that:

$$v_{II} = \sqrt{\frac{2GM}{R}}\,. \tag{21}$$

The difference between the first and the second cosmic velocities is the factor $\sqrt{2} = 1.4142\ldots$.

The third cosmic velocity

The third cosmic velocity is the velocity V_{III} which must be impressed to a body on the Earth so that it can escape from the solar system.
In fact, even if a velocity equal to the second cosmic velocity were impressed to a body of the solar system, it would not be able to go till the infinite because it would enter into elliptic orbit round the Sun, whose mass is enormous if compared to those of the other planets.

$$V_{III} = 16.7\,\mathrm{km/s}$$

At the surface of the Earth it equals approx. 42 km/s, but The Earth circuses around the Sun, so the smaller velocity is enough. o

The fourth cosmic velocity

The fourth cosmic velocity is the escape velocity from our galaxy - the Milky Way. It corresponds to about 320 kilometers per second. If one takes into account that the solar system revolves around the center of galaxy – it is enough that

$$v_{IV} \sim 130\,\mathrm{km/s}$$

It is important to bear in mind that these cosmic velocities are idealized values. For instance they do not take into account the loss of speed due to air resistance when a rocket is launched. Moreover, the values mentioned above are specific to the Earth and our solar system, and they do not apply to other parts of the universe.

The end? :-(

The end of the lecture 5