Physics for Computer Science Students
Lecture 12

Einstein's relativity theory

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Introduction

Einstein's relativity theory!

- Where it comes from?
- Paradoxes of the XIX century physics.
- Have we a solution?

Introduction

Reference system

The fixed reference system has to be defined in order to define the velocity of an arbitrary object.

Examples:

- wave in a line
- relative velocities of a passenger in a train with respect to:
  - rail-coach: $v = 0$ km/hour
  - railway station: $v = 100$ km/hour
  - middle of the Earth: $v = 1\,600$ km/hour
  - middle of the Sun: $v = 110\,000$ km/hour
  - middle of the Galaxy: ???

  Which one is the true velocity?

Very important questions: does exist the fixed reference system? Does exist the absolute reference system?

A.A. Michelson and E.W. Morley experiment (1887)

Light is a wave, so 'something' has to exists what can vibrate. This 'something' was called the ether (*phil.* according to ancient Greek philosophers it is a medium filling the Kosmos and defines as a primal matter, the fifth source of life (the rest four: fire, water, soil and air).

A.A. Michelson[a] and E.W. Morley were looking for the absolute reference system with the help of **interferometer** (see Fig. 1).

The monochromatic light beam id divided into two mutually perpendicular perpendicular beams. After reflection from the mirrors they come back to the place they were divided and the interference takes place. The result of the final picture depends on the difference of the mean velocities in the both directions.

[a]Albert A. Michelson was born in 1852 in Strzelno, Poland. When he was 2 years old, his parents have emigrated to USA. Michelson was the first american Nobel prize winner in physics.

Michelson-Morley experiment

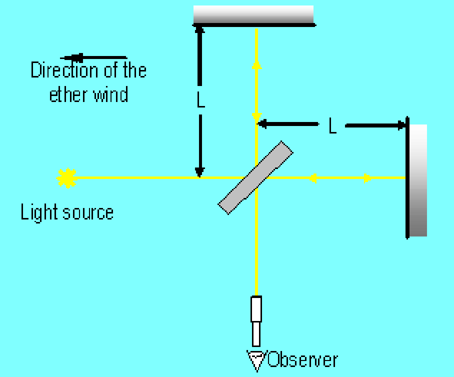

Fig. 1: Schema of the Michelson-Morley experiment

Michelson-Morley experiment

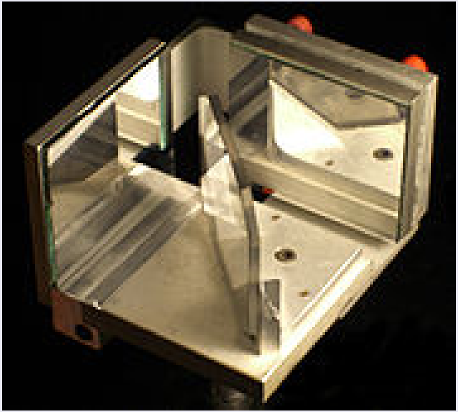

Fig. 2: Michelson-Morley interferometer

Michelson-Morley experiment

The interferometer was positioning in such a way, that one of its arms was parallel to the direction of motion of the Earth around the Sun. After half a year the order of arms was changed. This experiment was made many times and by other scientists too. The result was always the same: negative. No interference fringes were observed.

Earth linear velocity with respect to Sun is 30 km/s, it is approx. 1% of the light velocity. The precision of the used instrument was sufficiently good to state that the velocity of light is different in different directions in the case the velocity of light is added to the velocity of Earth, as it is observed for all classical phenomena.

Michelson-Morley experiment

Let us assume that Earth travels with the velocity v with respect to the ether. For the observer on Earth the direction of the ether wind is directed to Earth. The beam of light traveling perpendicularly to the direction of motion of Earth runs a little bit against the wind, but the ether wind takes it with and finally in the Michelson-Morley interferometer this beam runs perpendicularly to the direction of the Earth motion. (see Fig. 3).

Michelson-Morley experiment

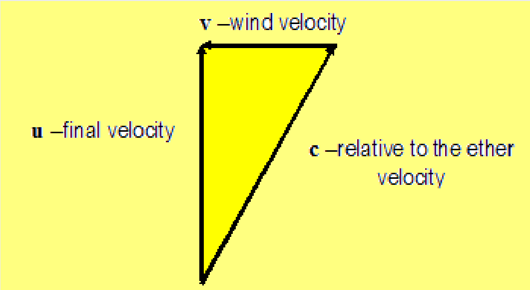

Fig. 3: Vectors of light velocities moving perpendicularly to the ether wind

Michelson-Morley experiment

It follows from Fig. 3, that the velocity of light with respect to the interferometer equals

$$u = \sqrt{c^2 - v^2}\,, \tag{1}$$

and time needed to cover this way in both direction equals

$$t_{\perp} = \frac{2L}{u} = \frac{2L}{\sqrt{c^2 - v^2}}\,. \tag{2}$$

Light send in the direction against the wind travels with the velocity c with respect to the ether and $c - v$ with respect to Earth. Time needed to reach the mirror equals $t_1 = L/(c - v)$. The velocity together with the wind equals correspondingly $t_2 = L/(c + v)$. The distance in both direction light covers in time $t_{||}$ equal to

$$t_{||} = \frac{L}{c - v} + \frac{L}{c + v} = \frac{2cL}{c^2 - v^2}\,. \tag{3}$$

Michelson-Morley experiment

having times $t_{\perp}$ i $t_{||}$ measured one is able to calculate the velocity of Earth with respect to the ether, or more precisely to detect this motion with the interferometer. In the case the time of travelling in different direction is different, the interference fringes should appear. They were not observed [4]. It occurred that
The Earth does not moves with respect to the ether.

Hendrik Antoon Lorentz and George Francis FitzGerald, in '90 of XIX century, independently, tried to explain the failure of the Michelson-Morley experiment by assuming that the length of the arm L of the interferometer along the direction of the Earth motion is shorter as in the rest situation. This phenomenon was later a part of the special relativity theory and the phenomenon is called the Lorentz-FitzGerald shortening.

The theory of relativity

The theory of relativity, or simply relativity, generally refers specifically to two theories of Albert Einstein: special relativity and general relativity. However, the word "relativity" is sometimes used in reference to Galilean invariance.
The term "theory of relativity" was coined by Max Planck in 1908 to emphasize how special relativity (and later, general relativity) uses the principle of relativity.

The theory of relativity

Special relativity

Special relativity is a theory of the structure of spacetime. It was introduced in Albert Einstein's 1905 paper *On the Electrodynamics of Moving Bodies*; however, the term was first used by Galileo Galilei in 1632 in his Dialogue concerning the *World's Two Chief Systems*. But Galileo's version was flawed: for example, he thought the spin of the Earth caused the tides. Special relativity is based on two postulates which are contradictory in classical mechanics:

- The laws of physics are the same for all observers in uniform motion relative to one another (Galileo's principle of relativity),
- The speed of light in a vacuum is the same for all observers, regardless of their relative motion or of the motion of the source of the light.

The theory of relativity

Special relativity

The resultant theory has many surprising consequences. Some of these are:

- Relativity of simultaneity: Two events, simultaneous for some observer, may not be simultaneous for another observer if the observers are in relative motion.
- Time dilation: Moving clocks are measured to tick more slowly than an observer's "stationary" clock.
- Length contraction: Objects are measured to be shortened in the direction that they are moving with respect to the observer.
- Mass-energy equivalence: $E = m\,c^2$, energy and mass are equivalent and transmutable.

The defining feature of special relativity is the replacement of the Galilean transformations of classical mechanics by the Lorentz transformations.

The theory of relativity

General relativity

General relativity is a theory of gravitation developed by Einstein in the years 1907–1915. The development of general relativity began with the equivalence principle, under which the states of accelerated motion and being at rest in a gravitational field (for example when standing on the surface of the Earth) are physically identical. The upshot of this is that free fall is inertial motion: In other words an object in free fall is falling because that is how objects move when there is no force being exerted on them, instead of this being due to the force of gravity as is the case in classical mechanics. This is incompatible with classical mechanics and special relativity because in those theories inertially moving objects cannot accelerate with respect to each other, but objects in free fall do so. To resolve this difficulty Einstein first proposed that spacetime is curved. In 1915, he devised the Einstein field equations which relate the curvature of spacetime with the mass, energy, and momentum within it.

The theory of relativity

General relativity

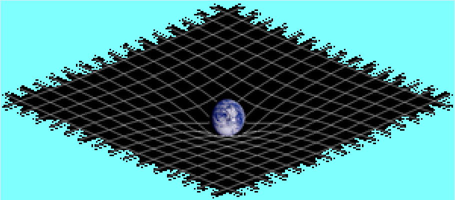

Fig. 4: Two-dimensional projection of a three-dimensional analogy of space-time curvature described in General Relativity.

The theory of relativity

General relativity

Some of the consequences of general relativity are:

- Time goes more slowly in higher gravitational fields. This is called gravitational time dilation.
- Orbits precess in a way unexpected in Newton's theory of gravity. (This has been observed in the orbit of Mercury and in binary pulsars).
- Rays of light bend in the presence of a gravitational field.
- Frame-dragging, in which a rotating mass "drags along" the space time around it.

Technically, general relativity is a metric theory of gravitation whose defining feature is its use of the Einstein field equations. The solutions of the field equations are metric tensors which define the topology of the spacetime and how objects move inertially.

Galilean transformations

The Galilean transformation is used to transform between the coordinates of two reference frames which differ only by constant relative motion within the constructs of Newtonian physics. This is the passive transformation point of view. The equations below, although apparently obvious, break down at speeds that approach the speed of light due to physics described by Einstein's theory of relativity.

Galileo formulated these concepts in his description of uniform motion. The topic was motivated by Galileo's description of the motion of a ball rolling down a ramp, by which he measured the numerical value for the acceleration of gravity, at the surface of the Earth.

Galileo 1638 *Discorsi e Dimostrazioni Matematiche, intorno á due nuoue scienze* 191 - 196, published by Lowys Elzevir (Louis Elsevier), Leiden, or Two New Sciences, English translation by Henry Crew and Alfonso de Salvio 1914, reprinted on pages 515-520 of *On the Shoulders of Giants: The Great Works of Physics and Astronomy*. Stephen Hawking, ed. 2002 ISBN 0-7624-1348-4

Galilean transformations

If the system $A' = (x', y', z')$ moves uniformly with the velocity v in the direction x with respect to the system $A = (x, y, z)$, and the axes of both coordinate systems are parallel to each other, so

$$x' = x - vt\,, \quad y' = y\,, \quad z' = z\,, \quad t' = t\,. \tag{4}$$

The distance between two points equals:

in system (x, y, z):

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}\,, \tag{5}$$

in system (x', y', z'):

$$d' = \sqrt{(x_1' - x_2')^2 + (y_1' - y_2')^2 + (z_1' - z_2')^2}\,, \tag{6}$$

It is seen that $d = d'$.

Galilean transformations

The velocity of light in the system (x, y, z) equals

$$c = \frac{dx}{dt}\,, \tag{7}$$

and in the system (x', y', z')

$$c' = \frac{dx'}{dt} = \frac{d(x - vt)}{dt} = c - v\,, \tag{8}$$

i.e.

$$c' \neq c\,. \tag{9}$$

In systems with Galilean transformations the velocity of light is different in the moving systems then in the rest systems – contradiction to the MM experiment.

Lorentz transformations

Invariance of the light velocity is ensured by the **Lorentz transformations**:

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}}\,, \qquad y' = y\,, \quad z' = z\,, \quad t' = \frac{t - v\dfrac{x}{c^2}}{\sqrt{1-\beta^2}}\,, \qquad (10)$$

$$x = \frac{x' + vt'}{\sqrt{1-\beta^2}}\,, \qquad y = y'\,, \quad z = z'\,, \quad t = \frac{t' + v\dfrac{x'}{c^2}}{\sqrt{1-\beta^2}}\,, \qquad (11)$$

$\beta = v/c$.

Lorentz transformations

Let at the instant $t = t' = 0$ the origins of the coordinate systems A and A' coincide. After a certain time t the light signal reaches the point (x, y, z) in A according to the condition

$$x^2 + y^2 + z^2 = c^2 t^2 \,, \tag{12}$$

which in the (x', y', z') co-ordinates in A' fulfills the condition (after the Lorentz transformations are made)

$$x'^2 + y'^2 + z'^2 = c^2 t'^2 \,. \tag{13}$$

It is seen that in both cases light travels with the velocity c.

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Relativity of simultaneity

Let in the system A there occur simultaneously at time $t = t_1 = t_2$ two events at points x_1 i x_2. In the system A', after the Lorentz transformation, one has

$$t'_1 = \frac{t_1 - \frac{v}{c^2}x_1}{\sqrt{1-\beta^2}}, \quad t'_2 = \frac{t_2 - \frac{v}{c^2}x_2}{\sqrt{1-\beta^2}}. \tag{14}$$

It is seen that $t_1 \neq t_2$: two simultaneous events in A are not simultaneous in A'.

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Time dilatation

Let the light signals are send from the point x of the system A in time intervals $\Delta t = t_2 - t_1$. In the moving co-ordinate system A', these intervals equal

$$\Delta t' = \frac{t_2 - t_1}{\sqrt{1-\beta^2}} = \frac{\Delta t}{\sqrt{1-\beta^2}} < \Delta t \, . \tag{15}$$

For the observer form the moving system the time intervals in the resting system are longer.

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Length shortening

One measures the length of a rod in a resting system A:
$d = x_2 - x_1$ at instant $t = t_1 = t_2$.

One measures the length of the same rod in a moving system A':
$d' = x'_2 - x'_1$ at instant $t' = t'_1 = t'_2$.

$$d' = x'_2 - x'_1 = (x_2 - x_1)\sqrt{1 - \beta^2} < d \,. \tag{16}$$

The length of a rod in the moving system is smaller.

If the system A' is treated as a resting one, the conclusion: **the rod has the greatest length in system in which it is at rest**.

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Addition of velocities

Point P' moves in a system A' with velocity u'. The system A' moves with respect to the immovable system A with the velocity v along the x axis. **What is the velocity of the point P' in the system A?**

$$u_i' = \frac{dx_i'}{dt'}\,, \quad u_i = \frac{dx_i}{dt}\,. \tag{17}$$

$$u_x = \frac{dx}{dt} = \frac{\frac{dx'}{dt} + v\frac{dt'}{dt}}{\sqrt{1-\beta^2}} = \frac{\left(\frac{dx'}{dt'} + v\right)\frac{dt'}{dt}}{\sqrt{1-\beta^2}} = \frac{u_x' + v}{\sqrt{1-\beta^2}}\,\frac{dt'}{dt}\,. \tag{18}$$

It follows from (10) that

$$\frac{dt'}{dt} = \frac{1 - \frac{v}{c^2}\frac{dx}{dt}}{\sqrt{1-\beta^2}} = \frac{1 - \frac{u_x v}{c^2}}{1-\beta^2}\,, \tag{19}$$

Addition of velocities

and finally

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}} \,. \tag{20}$$

For other co-ordinates of velocity:

$$u_y = \frac{dy}{dt} = \frac{dy'}{dt} = \frac{dy'}{dt'} \cdot \frac{dt'}{dt} = \frac{u'_y \sqrt{1-\beta^2}}{1 + \frac{v u'_x}{c^2}} \,, \tag{21}$$

$$u_z = \frac{u'_z \sqrt{1-\beta^2}}{1 + \frac{v u'_x}{c^2}} \,. \tag{22}$$

It follows from (20) that the resultant of two velocities is smaller then the sum of these two velocities. In particular, if $u'_x = c$ then $u_x = c$, i.e. the maximal velocity of electromagnetic waves in vacuum.

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Invariance of physical laws

Albert Einstein's postulate:

all physical laws are invariant with respect to the Lorentz transformations

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Equivalence of mass and energy

Newton's second law of dynamics

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{v}) = m\frac{d\mathbf{v}}{dt} + \mathbf{v}\frac{dm}{dt}\,. \tag{23}$$

Work of this force on the distance ds equals

$$F\,ds = m\,v\,dv + v^2\,dm\,. \tag{24}$$

After differentiation

$$m = \frac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}\,, \tag{25}$$

m_0 – rest mass.

Equivalence of mass and energy

One has

$$dm = \frac{mv\,dv}{c^2 - v^2}\,, \tag{26}$$

i.e.

$$F\,ds = dm(c^2 - v^2) + v^2 dm = c^2 dm = d(m\,c^2)\,. \tag{27}$$

The elementary work causes the decrement of the potential energy, so

$$-\,dU = d(m\,c^2)\,. \tag{28}$$

After integration

$$E = mc^2 + U = \frac{m_0\,c^2}{\sqrt{1 - \frac{v^2}{c^2}}} + U = \text{const}\,, \tag{29}$$

E – total energy of the moving body in the force field.

Equivalence of mass and energy

Developing into series

$$E = m_0 c^2 + \left(\frac{1}{2} m_0 v^2 + \frac{3}{8} m_0 \frac{v^4}{c^4} + \dots \right) + U\,. \tag{30}$$

Taking as a reference point $U = 0$, one has **equivalence of mass and energy law**

$$E = m\,c^2\,. \tag{31}$$

It is the energy conservation law in the field of the conservative forces.
It follows from (30)

$$\begin{aligned} m = \frac{E}{c^2} &= m_0 + \frac{1}{c^2} \left(\frac{1}{2} m_0 v^2 + \frac{3}{8} m_0 \frac{v^4}{c^2} + \dots \right) + \frac{U}{c^2} \\ &= m_0 + m_{kin} + m_{pot} = \text{const}\,. \end{aligned} \tag{32}$$

Conclusions

It was shown by Neil Ashby, that the GPS can not work properly, if the relativistic corrections are not taken into account. Already after 24 hours the error in the positioning is 18 km if the influence of the gravitational field on the passage of time is neglected.

Nevertheless the Einstein's relativity theory will be changed, it is not eternal!

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The end? :-(

The end of the lecture 12