Physics for Computer Science Students
Lecture 11

Optics

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Introduction

What is the real nature of light? Does it consist of elementary particles or waves?

	Waves	Quants
1	Interference	Photoelectricity
2	Diffraction	Color
3	Refraction	

Is this a physical question?

Introduction

Christiaan Huygens

Fig. 1: Christiaan Huygens (1629 - 1695)

Christian Huygens (1629 - 1695) – was a Dutch mathematician who patented the first pendulum clock, which greatly increased the accuracy of time measurement. He laid the foundations of mechanics and also worked on astronomy and probability. Around 1654 he devised a new and better way of grinding and polishing lenses. Using one of his own lenses, Huygens detected, in 1655, the first moon of Saturn. In this same year he made his first visit to Paris. He informed the mathematicians in Paris including Boulliau of his discovery and in turn Huygens learnt of the work on probability carried out in a correspondence between Pascal and Fermat. On his return to Holland Huygens wrote a small work *De Ratiociniis in Ludo Aleae* on the calculus of probabilities, the first printed work on the subject.

Introduction

Christiaan Huygens

In 1678 his *Traité de la lumiere* appeared, in it Huygens argued in favour of a wave theory of light. Huygens stated that **an expanding sphere of light behaves as if each point on the wave front were a new source of radiation of the same frequency and phase**. However his health became even more unreliable and he became ill in 1679 and then again in 1681 when he returned to the Hague for the last time.
In England Huygens met Newton, Boyle and others in the Royal Society. It is not known what discussions went on between Huygens and Newton but we do know that Huygens had a great admiration for Newton but at the same time did not believe the theory of universal gravitation which he said
appears to me absurd
In some sense of course Huygens was right, how can one believe that two distant masses attract one another when there is nothing between them, nothing in Newton's theory explains how one mass can possible even know the other mass is there. Writing about Newton and the Principia some time later Huygens wrote: *I esteem his understanding and subtlety highly, but I consider that they have been put to ill use in the greater part of this work, where the author studies things of little use or when he builds on the improbable principle of attraction.*

Introduction

Christiaan Huygens

Huygens scientific achievements can be summed up as follows: He combined Galileo's mathematical treatment of phenomena with Descartes' vision of the ultimate design of nature. Beginning as an ardent Cartesian who sought to correct the more glaring errors of the system, he ended up as one of its sharpest critics. The ideas of mass, weight, momentum, force, and work were finally clarified in Huygens' treatment of the phenomena of impact, centripetal force and the first dynamical system ever studied – the compound pendulum.

http://www.maths.tcd.ie/pub/HistMath/People/Huygens/RouseBall/RB_Huygens.html
http://www.gap-system.org/history/Biographies/Huygens.html

Definitions

Units of optical quantities

Candela (cd): luminous intensity (of a typical common candle) in a given direction of a source that emits monochromatic radiation of frequency 540×10^12 hertz and that has a radiant intensity in that direction of $1/683$ watt per steradian.

Steradian (sr): a such conical in shape solid angle that the area A of the subtended portion of the sphere is equal to r^2, where r is the radius of the sphere.

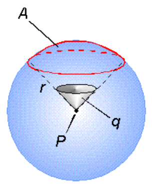

Fig. 2: Definition of Steradian

Definitions

Units of optical quantities

Quantity	Notation	Description
luminous intensity	[cd]	candela
illuminance	[cd/m^2]	candela per square meter
luminous flux	[lm]	lumen: lm = cd·sr
luminous quantity	[lm·s]	lumenosecond: lm·s

Geometrical optics

Fundamental assumptions of geometrical optics

The ray model of light:

1. light travels in straight-line paths called light rays; in the corpuscular (quantum) theory the paths of photons; in the wave theory in the direction perpendicular to the wave front;
2. dimensions of obstacles and holes are big as compared to the wave length;
3. Fermat's principle: light travels between two points along the path that requires the least time, as compared to other nearby paths.

Shadows from the light sources

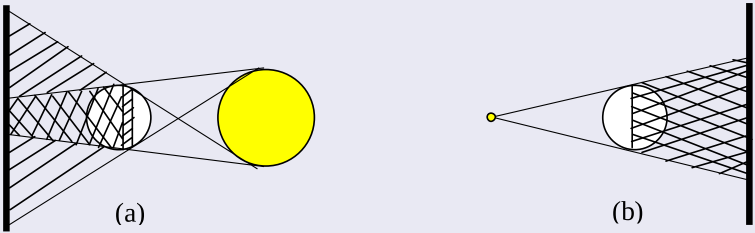

Fig. 3: Shadows from the light sources: (a) light source dimension is bigger then the dimension of an obstacle, the penumbras (half-lights) are seen; (b) light source dimension is smaller then the dimension of an obstacle

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Reflection and dispersion of light

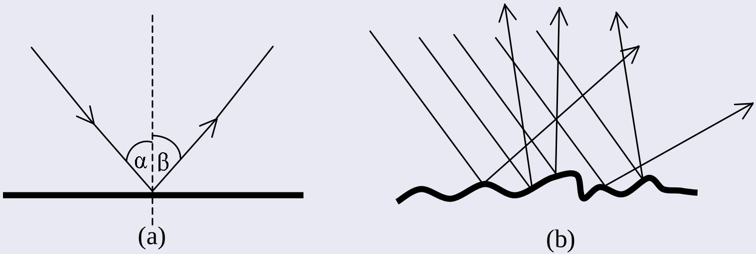

Fig. 4: Light striking a surface: (a) smooth surface; (b) rough surface

Light reflection

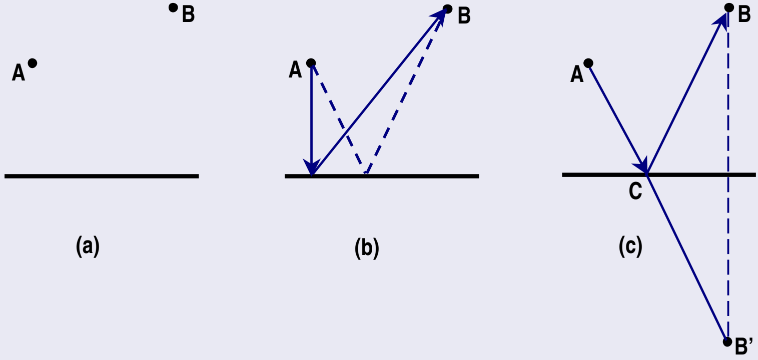

Fig. 5: Light striking a surface: (a) two points A i B over the reflecting surface; (b) possible trajectories of light rays between points A i B after reflection; (c) applying the Fermat's principle

Light reflection

Question 1: how can light travel from point A to pont B? (see Fig. 5(a))
Answer 1: along the line connecting these points .

Question 2: and how it travels when the mirror reflection occurs on the way?? (see Fig. 5(b))
Answer 2: it is a little more complicated – one of possible trajectories is shown as a solid line; in the case of the trajectory shown as the dashed lined the total way will be shorter and time needed to cover that trajectory will be shorter too.

Question 3: where there is the real point of reflection from the mirror? (see Fig. 5(c))
Answer 3: this is the point C.

Light reflection

Analysis of Fig. 5(c) gives the following conclusion:

the angle of reflection equals the angle of incidence

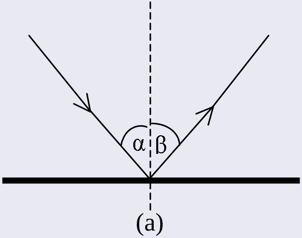

Fig. 6: Light striking the smooth surface

so in Fig. 6 the angle α equals the angle β.

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Light refraction

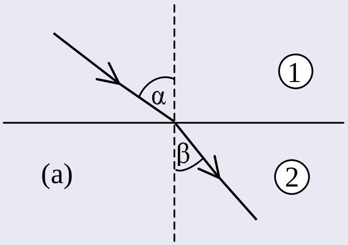

Fig. 7: Light refraction: (a) ray bends toward the normal ($\alpha > \beta$) ; (b) ray bents away from the normal ($\alpha < \beta$)

Light refraction

The light ray traveling in the material medium 1 with the velocity v_1 and striking the boundary with the other material medium 2, in which its velocity is v_2 (e.g. air and water), is partially reflected and partially refracted (the direction of propagation is changed, see. Fig. 7).

- angle β angle of refraction– between the light ray direction and normal to the surface

Important: incident ray, normal to the surface and refracted ray there are in one plane.

Refraction law (Snell's law, 1621)

$$\frac{\sin\alpha}{\sin\beta} = \frac{v_1}{v_2} = \frac{n_2}{n_1} = n_{2,1}\,, \tag{1}$$

$$n_1 = \frac{c}{v_1}\,, \qquad n_2 = \frac{c}{v_2}\,, \tag{2}$$

n_i – respective index of refraction in medium i, c – light velocity in vacuum. In vacuum refraction index $n_0 = 1$, so in the situation vacuum - medium with the refraction index n, one has

$$\sin\alpha = n\,\sin\beta\,, \tag{3}$$

n – absolute refraction index.

Prism: light refraction

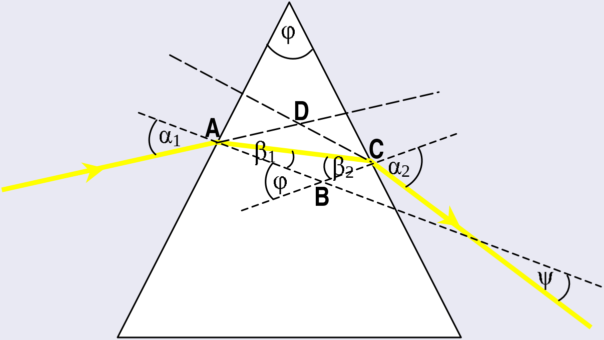

Fig. 8: Light ray refraction in a prism

Light ray refraction in a prism

Prism – equally sided triangle transparent body cuted out from a parallelepiped
φ – prism angle
ψ – deflection angle
n_1 – prism refraction index
n_2 – medium refraction index
(in general $n_2 < n_1$)

$$n_2 \sin\alpha_1 = n_1 \sin\beta_2\,, \quad n_1 \sin\beta_2 = n_2 \sin\alpha_2\,. \tag{4}$$

From $\triangle$ABC and $\triangle$ACD $\rightsquigarrow$

$$\varphi = \beta_1 + \beta_2\,, \quad \psi = (\alpha_1 - \beta_1) + (\alpha_2 - \beta_2) = (\alpha_1 + \alpha_2) - (\beta_1 + \beta_2)\,, \tag{5}$$

$$\psi = \alpha_1 + \alpha_2 - \varphi\,. \tag{6}$$

Light ray refraction in a prism

For small α_i and β_i (in radians)

$$\alpha_1 = \frac{n_1}{n_2}\beta_1 \,, \quad \alpha_2 = \frac{n_1}{n_2}\beta_2 \,, \tag{7}$$

and then

$$\psi = \frac{n_1}{n_2}\varphi - \varphi = \left(\frac{n_1}{n_2} - 1\right)\varphi \,. \tag{8}$$

For air $n_2 \approx 1$, so

$$\psi = (n_1 - 1)\,\varphi \,. \tag{9}$$

Light ray refraction in a prism

Determining the refraction index of a prism

The minimal deflection of the ray occurs for $\alpha_1 = \alpha_2 = \alpha$ and obviously $\beta_1 = \beta_2 = \beta$, i.e. $2\beta = \varphi$ (symmetric trajectory of rays)

$$\psi_{min} = 2\alpha - 2\beta = 2\alpha - \varphi\,. \tag{10}$$

One knows: ψ_{min} and φ, and it follows $\rightsquigarrow$

$$\alpha = \frac{\psi_{min} + \varphi}{2}\,, \quad \beta = \frac{\varphi}{2}\,, \tag{11}$$

$$n_1 = n = \frac{\sin\alpha}{\sin\beta} = \frac{\sin\frac{\psi_{min}+\varphi}{2}}{\sin\frac{\varphi}{2}}\,. \tag{12}$$

For small φ one has $\psi = (n-1)\,\varphi\,$,

$$n_{prism} = \frac{\psi_{min}}{\varphi} + 1\,. \tag{13}$$

Dispersion of light in prism

Newton (1666): white light (Sun light) is a mixture of color rays. The white light ray passing through a prism is broken down into its constituent colors.

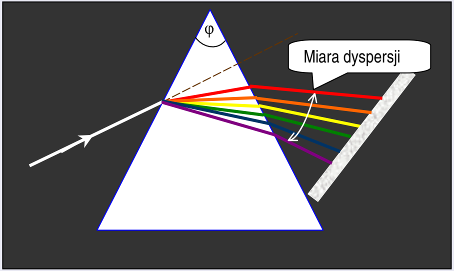

Fig. 9: Dispersion of light

The color of light rays depends on its wave length ⇝ to every wave length corresponds different refraction index.

Dispersion of light in prism

Measure of the prism dispersion ability: difference of refraction indices for red and violet rays (mean prism dispersion.)

Measure of the prism refraction ability: value of the refraction index for the yellow ray.

Dispersion of light in prism

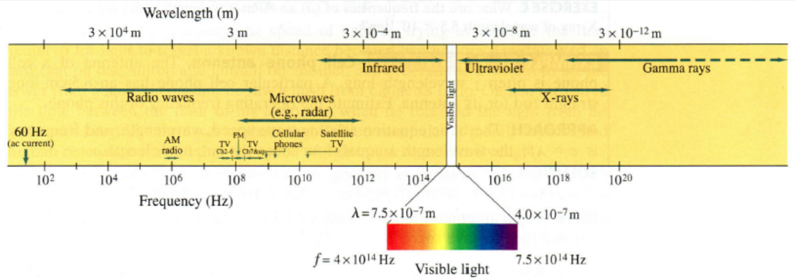

Fig. 10: Spectrum of electromagnetic waves

Dispersion of light in prism

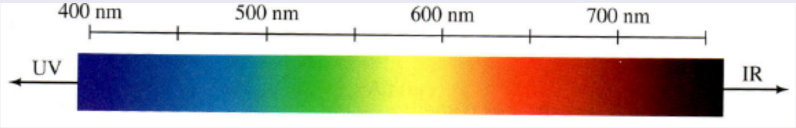

Fig. 11: Spectrum of visible light

Dispersion of light in prism

Fig. 12: Rainbow seen from Waikiki

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Total internal reflection

When light ray travels from a medium with a greater optical density (water, glass) with the refraction index n_1 into the medium with a smaller optical density (air) with the refraction index n_2, so according to the Snell's principle

$$n_1 \sin\alpha = n_2 \sin\beta \,. \tag{14}$$

$n_2 < n_1$, so $\sin\alpha < \sin\beta$, i.e. $\alpha < \beta$. For greater α's correspond greater β's, but always $\alpha < \beta$. Angle α, for which refraction angle $\beta = 90^\circ$ is called **the limit angle**.
For the angles greater then the limit angle the **total internal reflection** occurs.
By the light transition from water ($n_1 = 1,33$) into air ($n_2 \approx 1$)

$$\sin\alpha_{gr} = \frac{1}{n_1} = 0,748 \,, \tag{15}$$

i.e. for water $\alpha_{gr} \sim 48^\circ 30'$.

Total internal reflection

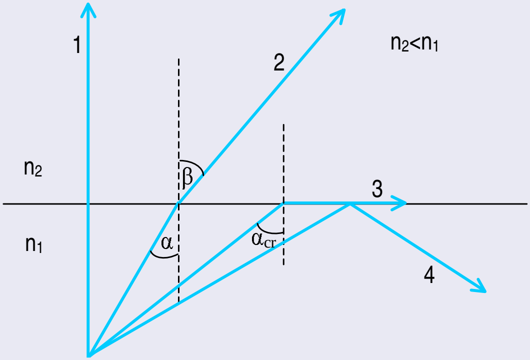

Fig. 13: Internal reflection

Total internal reflection

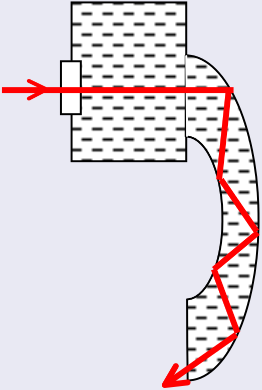

Fig. 14: Light reflected totally at the interior surface of a transparent plastic fiber

Total internal reflection

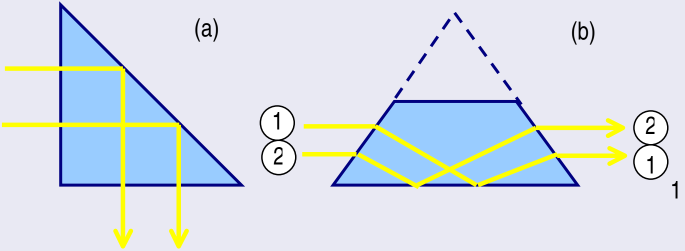

Fig. 15: Total internal reflection in a prism: (a) change of the direction of rays at 90°; (b) change of the order of rays (Amici prism)

Slab with two parallel faces

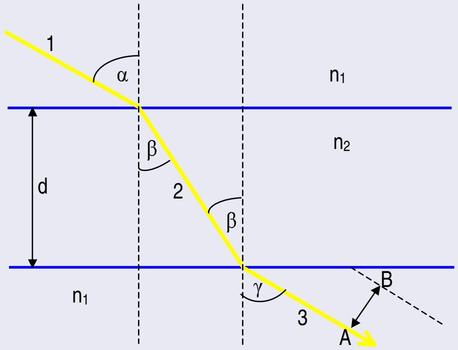

Fig. 16: Light refraction in a lab with two parallel faces

Slab with two parallel faces

From Snell's principle

$$\begin{aligned} n_1 \sin\alpha &= n_2 \sin\beta\,, \\ n_2 \sin\beta &= n_1 \sin\gamma\,. \end{aligned} \tag{16}$$

It is seen that $\alpha = \gamma$. Displacement of a ray AB depends on the slab thickness d, incident angle α and on the refraction indices n_1 i n_2

$$\mathrm{AB} = d\,\frac{\sin(\alpha-\beta)}{\cos\beta}\,. \tag{17}$$

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Lenses

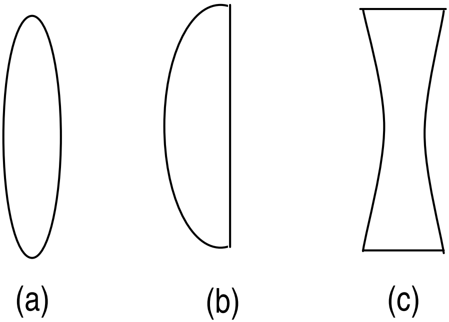

Fig. 17: Examples of lenses: (a) i (b) converging lenses; (c) diverging lenses

Magnifying glass

Converging lens

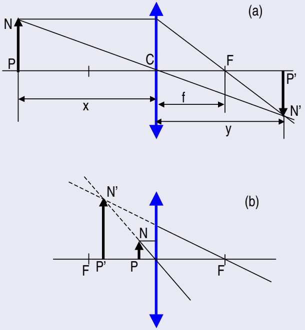

Fig. 18: Examples of applications of converging lens: (a) object in the distance greater then the len's focal length; (b) object in the distance smaller then the len's focal length;

Magnifying glass

Diverging lens

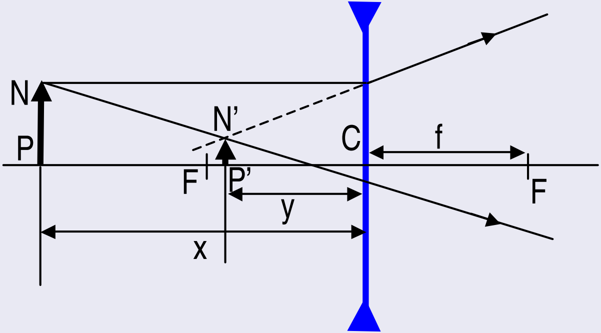

Fig. 19: Trajectories of light rays in diverging lens

Lens power

Diopter (D, dpt., δ) – unit of the optical device power, accepted by SI system.

$$1\mathrm{D} = \frac{1}{\mathrm{m}}$$

Positive values define converging lenses, and negative values – diverging lenses.
Applied in glasses.

Lens power

One has

$$D = \frac{1}{f} = \left(\frac{n_s}{n_0} - 1\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{18}$$

f – focal length (distance between focus and middle of a lens)
n_s – absolute refraction index of a lens
n_0 – absolute refraction index of an environment
r_1, r_2 – curvature radii of lenses (for planoconcave or planoconvex one of radii $= \infty$)
where $n = \frac{c}{v}$ – absolute refraction index
c – light velocity in vacuum
v – light velocity in medium

Magnifying glass

Magnifying glass

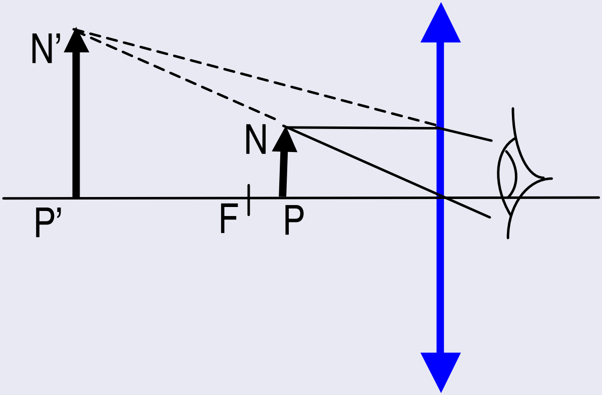

Fig. 20: Trays of light rays in magnifying glass

Magnifying glass

Magnifying glass

Magnifying glass: lens with a small focal length f (with big power $(1/f)$. PN – object and P'N' – image. It follows from Fig. 20) that magnification equals

$$w = \frac{\mathrm{P'N'}}{\mathrm{PN}} = \frac{d}{x}\,. \tag{19}$$

From the lens equation

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{f}\,, \tag{20}$$

$$x = \frac{fd}{d-f}\,. \tag{21}$$

$$w = \frac{d}{x} = \frac{d-f}{f} = \frac{d}{f} - 1 \approx \frac{d}{f}\,. \tag{22}$$

Magnifying glass

Lupa

Image appears on the same side of a lens as object, so $x > 0$, $f > 0$, but $d < 0$ and $w < 0$, i.e. an image is virtual because $|w| > 1$. The image is so greater as the focal length is smaller. E.g. magnifying glass with the focal length $f = 5$ cm gives the six-times magnification

$$|w| = \left| \frac{-25}{5} - 1 \right| = 6\,.$$

In order to have the 11-times magnification one has to use the magnifying glass with focal length $f = 2,5$ cm.

The end? :-(

The end of the lecture 11