# Physics for Computer Science Students
# Lecture 10

## Electrodynamics


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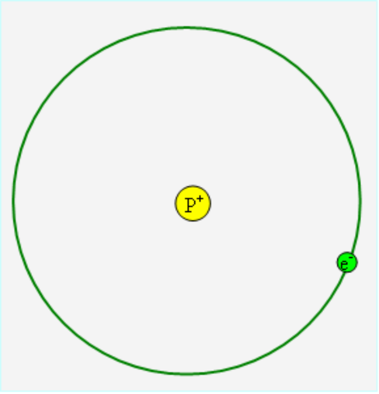


Fig. 1: Model of the hydrogen atom

# Contents

# Contents

# Contents

# Contents

# Definitions

## Units of electromagnetic quantities

| Name | Notion | Description |
|---|---|---|
| Electric charge | [C] | 1 A·s |
| Elementary electric charge | [e] | $1.60217733 \cdot 10^{-19}$[A·s] |
| Electric current | [A] | fundamental quantity, direct electric current in two ∞ - long straight parallel electric wirings with ∞ - small cross sections in vacuum and in distance 1m from each other and acting on one another with the force $2 \cdot 10^{-7}$ [N/m] |
| Intensity of electric field | [E] | $[E] = \frac{[N]}{[C]} = \left[\frac{kg\ m\ s^{-2}}{A \cdot s}\right]$ |
| Electron (rest) mass | $m_e$ | $9.1093897 \cdot 10^{-31}$ kg |
| Proton (rest) mass | $m_p$ | $1.6726231 \cdot 10^{-27}$ kg |

# Electrostatics

The science about the interactions of the electric charges, being in the rest with respect to the chosen co-ordinate system.

There exist two types of the electric charge only: negative and positive. The charges of the same type repulse each other and the charges of the different types attract each other.

**Electric charge conservation law**

The algebraic sum of the electric charges in the isolated system is constant. In the electrically neutral system the numbers of the positive and negative charges are equal.

# Electrostatics

The total electric charge of an arbitrary body is the sum of its elementary charges.

## Electron

It is the smallest and the stabile elementary particle having the unit **negative** electric charge. The mass of electron equals $9,1 \cdot 10^{-28}$g.

## Proton

It is the smallest and the stabile elementary particle having the unit **positive** electric charge. The mass of proton equals $1,67 \cdot 10^{-24}$g.

# Coulomb's law

The force $\mathbf{F}_{12}$ of the electrostatic interactions of two point electric charges $q_1$ and $q_2$ equals:

$$\mathbf{F}_{12} = k\,\frac{q_1\,q_2}{r^2}\,\frac{\mathbf{r}_{12}}{r}\,, \qquad (1)$$

$\mathbf{r}_{12} = -\mathbf{r}_{21}$;
$k$ – coefficient of proportionality, depends on the properties of a medium;
$k = \dfrac{1}{4\pi\varepsilon_0\varepsilon}$ – in MKSA (SI) system;
$k = \dfrac{1}{\varepsilon}$ – in cgs system;
$\varepsilon_0 = 8.5 \cdot 10^{-12}$ [C $^2$/N $\cdot$ m$^2$] – permittivity of free space;
$\varepsilon$ – relative electric permittivity of a medium, it shows how many times the interaction force of two charges in a medium is smaller as compared with the the interaction force in free space (vacuum).

# Electromagnetic field

In the **macroscopic** theory of electromagnetic field (EMF) the **microscopic** (atomic) structure of matter is not taken into account – the continuous distribution of matter is assumed. The properties of EMF at the every point of a body are defined by the following material parameters:

- $\varepsilon$ – electric permittivity
- $\mu$ – magnetic permittivity
- $\sigma$ – proper electric conductivity

It is mostly assumed that they are constant, do not depend on the state of a field.

It is also assumed that:

- $\rho = \rho(\mathbf{x}, t)$ – gęstość ładunku elektrycznego
- $\mathbf{J} = \mathbf{J}(\mathbf{x}, t)$ – gęstość prądu elektrycznego

It is a conscious limitation of the theory.

# James Clerk Maxwell (1831-1879)

## formulated his theory in 1864

It is the synthesis of the all known before electrodynamic laws, and simultaneously a generalization of them. From the one point of view it describes electrostatics (the interaction of charges being in rest), light and radio waves, and etc.

One can say, instead of saying that the point charge $q_1$ acts on the point charge $q_2$ with the force given by Coulomb's equation (Charles Coulomb, 1736 - 1806, formulated his theory in 1785), that the charge $q_1$ creates the EMF, which acts on the point charge $q_2$, and $q_2$ also creates a field acting on $q_1$. in the case the charges are fixed, the Maxwell's theory does not introduce any new physical aspects. The situation changes dramatically when the charges are moving – field starts to play a very important rôle and the Coulomb' law is not sufficiently satisfactory.

# Velocity of light $c \cong 3 \cdot 10^8 \mathrm{m/\ s}$

changing the position of charge $q_1$ influences on the the state of charge $q_2$ after a certain time: the field gains the physical meaning – charge $q_1$ interacts with field, and later field interacts with charge $q_2$.

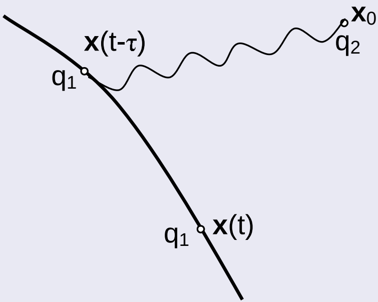


Fig. 2: Field interaction of charges

# Maxwell equations

Maxwell equations have some different formulations, and always use the two fundamental fields:

- electric field intensity**E**, [V/m] and
- magnetic field intensity**H** [A/m].

If moreover these fields there are in the material medium,two additional fields appear: :

- electric flux density **D**, [C/$m^2$] and
- magnetic flux density **B** [Weber/$m^2$].

These fields have their sources: electric field – electric charges, and magnetic field – electric currents. Electric currents not only excites fields, the are also created under the influence of fields. In conductors they can also come into being under the influence of differences in charge concentrations and temperature differences.

# Maxwell equations

Notion 'point charge' is to be understood analogously like the 'material point' in mechanics. The density of point charges is impossible to describe using the continuous function $\rho = \rho(\mathbf{x})$. This difficulty is omitted by introduction the $\delta$-Dirac functional, defined as follows

$$\varphi(a) = \int_{-\infty}^{\infty} \delta(x)\varphi(x+a)dx \,. \tag{2}$$

Sometimes there are charge distributions along curves or surfaces

$$\rho(\mathbf{x}) = \int_C \lambda(s)\delta^3(\mathbf{x} - \mathbf{x}'(s))ds \,, \tag{3}$$

$\mathbf{x}' = \mathbf{x}'(s)$ – parametric description of a curve, $\lambda(s)$ – linear density of charge.

# Maxwell equations

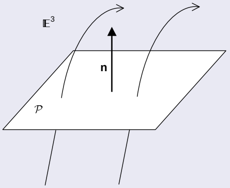


Fig. 3: Flow of electric current through the plane $\mathcal{P}$

Electric current flowing through the plane $\mathcal{P}$ is given in form

$$J = \iint\limits_{\mathcal{P}} \mathbf{J} \cdot \mathbf{n} d\sigma \,. \tag{4}$$

# Maxwell equations

Vector of the current density $\mathbf{J}$ is often shown in the form

$$\mathbf{J} = \mathbf{J}^{(p)} + \mathbf{J}^{(z)}\,, \tag{5}$$

$\mathbf{J}^{(p)} = (\sigma)\mathbf{E}$ – conductive current, $\mathbf{J}^{(z)}$ – independent external currents caused e.g. by the difference of charge concentrations $\rho$ or the difference of temperature $T$

$$\mathbf{J}^{(z)} = -\alpha\,\mathrm{grad}\,\rho\,, \qquad \mathbf{J}^{(z)} = -\beta\,\mathrm{grad}\,T\,, \tag{6}$$

$\alpha$ – diffusion coefficient, $\beta$ – thermal coefficient.

# Contents

# Maxwell equations – integral form

## 1. Michael Faraday (1791-1867)

he established experimentally the electromagnetic induction law

$$\oint_{\partial S} \mathbf{E}(\mathbf{x}, t) d\mathbf{x} = -\frac{d}{dt} \iint_S \mathbf{B} \cdot \mathbf{n} \, d\sigma \; = -\frac{d}{dt} \Phi \,, \qquad (7)$$

rhs – velocity of changes of the magnetic induction flux flowing through the plane $S$,
lhs – electromotoric force corresponding to the changes of the magnetic induction flux
If the integration circuit replace by the closed electric circuit – the electric current will flow with the intensity depending on the electromotoric force and the resistance of the circuit. The direction of the induced current is such, that the produced EM field counteracts the changes of the induction flux $\Phi$ – method of the measurement of the electric field.

# Maxwell equations – integral form

## 2. Andre Marie Ampère (1775-1836), Hans Christian Oersted (1777-1851)

$$\oint_{\partial S} \mathbf{H}(\mathbf{x}, t) d\mathbf{x} = \frac{d}{dt} \iint_S \mathbf{D} \cdot \mathbf{n}\, d\sigma + \iint_S \mathbf{J} \cdot \mathbf{n}\, d\sigma\,, \tag{8}$$

It is the generalization of the Ampère's-Oersted's law and contains the most important element introduced by Maxwell to electrodynamics, namely $\dot{\mathbf{D}}$.

The total current

$$\mathbf{C} = \dot{\mathbf{D}} + \mathbf{J}\,, \tag{9}$$

**C** – definition introduced by Maxwell (*current*): electromotoric force is generated not only by the conductivity current **J**, but also by the changing of electric flux density – in the alternate electric field the change in time the electric induction flux flowing through the plane $S$ has the same result as flow of the electric current..

# Maxwell equations – integral form

## 3. Carl Friedrich Gauß(1777-1855) – electric law

$$\iint_{\partial\Omega} \mathbf{D} \cdot \mathbf{n} \, d\sigma = \iiint_{\Omega} \rho \, d\omega = Q \,, \tag{10}$$

the total charge $Q$ in a certain area $\Omega$ is equal to electric induction flux $\mathbf{D} \cdot \mathbf{n}$ flowing through the closed surface $\partial\Omega$.

# Maxwell equations – integral form

## 4. Carl Friedrich Gauß(1777-1855) – magnetic law

$$\iint_{\partial\Omega} \mathbf{B} \cdot \mathbf{n}\, d\sigma = 0\,, \tag{11}$$

in the nature there are not the single magnetic poles.

# Contents

# Constitutive equations

In isotropic media

$$\begin{aligned} \mathbf{D}(\mathbf{x},t) &= \varepsilon(\mathbf{x},t)\,\mathbf{E}(\mathbf{x},t)\,, \\ \mathbf{B}(\mathbf{x},t) &= \mu(\mathbf{x},t)\,\mathbf{H}(\mathbf{x},t)\,, \\ \mathbf{J}(\mathbf{x},t) &= \sigma(\mathbf{x},t)\,\mathbf{E}(\mathbf{x},t)\,, \quad \text{(George Ohm's law, 1787-1854)} \end{aligned} \tag{12}$$

- $\varepsilon(\mathbf{x},t)$ – electric permeability of medium, $[\varepsilon] = \dfrac{[D]}{[E]} = \dfrac{\mathrm{F}}{\mathrm{m}}$, Farad $= \mathrm{C/V}$
- $\mu(\mathbf{x},t)$ – magnetic permeability of medium, $[\mu] = \dfrac{[B]}{[H]} = \dfrac{\mathrm{H}}{\mathrm{m}}$, Henr $= \mathrm{V \cdot s/A}$
- $\sigma(\mathbf{x},t)$ – proper conductivity of medium, $[\sigma] = \dfrac{[J]}{[E]} = \dfrac{\mathrm{S}}{\mathrm{m}}$, Siemens $= \mathrm{A/V}$

# Constitutive equations

Particulary in vacuum

- $\varepsilon_0 = \dfrac{10^7}{4\pi c^2}$F/m – electric permeability of vacuum
- $\mu_0 = 4\pi \cdot 10^{-7}$H/m – magnetic permeability of vacuum

It is seen that

$$c = \frac{1}{\sqrt{\varepsilon_0\,\mu_0}}\,, \tag{13}$$

$c = 2,9978 \cdot 10^8$m/s – light velocity in vacuum.

In material medium

$$\varepsilon = \varepsilon_0\,\varepsilon_w\,, \qquad \mu = \mu_0\,\mu_w\,, \tag{14}$$

$\varepsilon_w$ – relative electric permeability , $\mu_w$ – relative magnetic permeability.

# Constitutive equations

$\varepsilon_w$, $\mu_w$ – dimensionless quantieties.

The are also in use

$$\chi = \varepsilon_w - 1\,, \qquad \kappa = \mu_w - 1\,, \tag{15}$$

$\chi$ – electric susceptibility, $\kappa$ – magnetic susceptibility.

## Dielectrics – non-conductive media $\varepsilon_w \geq 1$

$$\mathbf{D} = \varepsilon_0 \varepsilon_w \mathbf{E} = \varepsilon_0 \mathbf{E} + \varepsilon_0 \chi \mathbf{E} = \varepsilon_0 \mathbf{E} + \mathbf{P}\,. \tag{16}$$

**P** – polarization vector. Phenomenon of polarization causes that in dielectrics electric field is a superposition of two fields:

1. external field, produced by the charges not connected with the dielectric;
2. field generated as a result of changes in dielectric by the external fields – i.e. the polarization of a medium.

# Constitutive equations

## Magnetyki

For almost all media $\mu_w \approx 1$.

- $\mu_w > 1$ – paramagnetics
- $\mu_w < 1$ – diamagnetics

$$\mathbf{B} = \mu_0 \mu_w \mathbf{H} = \mu_0 \mathbf{H} + \mu_0 \kappa \mathbf{H} = \mu_0 \mu_w \mathbf{H} + \mathbf{M}\,. \tag{17}$$

**M** – magnetization vector.

Third group of media – **ferromagnetics**, in frames of phenomenological theory it is difficult to describe them, because the discrete structure of the material is not taken into account (domains).

# Contents

# Maxwell equations – differential form

Differential form of Maxwell equation follows from the application of the Stokes and Gauss-Ostrogradzki theorems.

$$
\begin{aligned}
\text{rot } \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} &= 0\,, \\
\text{rot } \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} &= \mathbf{J}\,, \\
\operatorname{div} \mathbf{D} &= \rho\,, \\
\operatorname{div} \mathbf{B} &= 0\,,
\end{aligned}
\tag{18}
$$

$$
\mathbf{D} = (\varepsilon)\mathbf{E}\,, \quad \mathbf{B} = (\mu)\mathbf{H}\,, \quad \mathbf{J} = (\sigma)\mathbf{E} + \mathbf{J}^{(z)}\,. \tag{19}
$$

## Descriptions

- **E** – vector of electric field intensity, $[E] = \dfrac{\mathrm{V}}{\mathrm{m}} = \dfrac{\mathrm{m}^2 \cdot \mathrm{kg}}{\mathrm{A} \cdot \mathrm{s}^3}$
- **D** – vector of the dielectric induction, $[\mathbf{D}] = \dfrac{\mathrm{A} \cdot \mathrm{s}}{\mathrm{m}^2} = \dfrac{\mathrm{C}}{\mathrm{m}^2}$
- **H** – vector of magnetic field intensity, $[\mathbf{H}] = \dfrac{\mathrm{A}}{\mathrm{m}}$
- **B** – vector of the magnetic induction, $[\mathbf{B}] = \dfrac{\mathrm{kg}}{\mathrm{A} \cdot \mathrm{s}^2} = 1$ tesla
- $\Psi$ – flux of the electric induction field, $[\Psi] = \mathrm{C} = \mathrm{A} \cdot \mathrm{s}$
- $\Phi$ – flux of the magnetic induction field, $[\Phi] = \dfrac{\mathrm{kg} \cdot \mathrm{m}^2}{\mathrm{A} \cdot \mathrm{s}^2} = 1$ weber

# EM fields independent on time

In electrodynamics the following fields are discussed, as a result of time dependence elektrodynamiki:

1. static EMF: $\mathbf{E} = \mathbf{E}(\mathbf{x})$, $\mathbf{H} = \mathbf{H}(\mathbf{x})$, $\rho = \rho(\mathbf{x})$, $\mathbf{J} = \mathbf{0}$
2. stationary EMF: all as upper, but $\mathbf{J} = \text{const} \neq \mathbf{0}$
3. quasi-stationary EMF: field changes in time very slowly and shifted current can be neglected, i.e. $\dot{\mathbf{D}} = 0$, but $\dot{\mathbf{B}} \neq 0$ and $\mathbf{J} \neq 0$
4. general case: there are inequalities $\dot{\mathbf{D}} \neq 0$, $\dot{\mathbf{B}} \neq 0$, $\mathbf{J} \neq \mathbf{0}$ and the full system of Maxwell equations has to be used.

# EM fields independent on time

## Static fields

| / | I | II |
|---|---|---|
| 1. | rot $\mathbf{E} = 0$ | rot $\mathbf{H} = 0$ |
| 2. | $\mathrm{div}\, \mathbf{D} = \rho$ | $\mathrm{div}\, \mathbf{B} = 0$ |
| 3. | $\mathbf{D} = \varepsilon \mathbf{E}$ | $\mathbf{B} = \mu \mathbf{H}$ |

I – electrostatic field, II magnetostatic field
They can be considered independently, but the transition to other inertial reference system gives the relation $\tilde{\mathbf{J}} = \mathrm{const} \neq \mathbf{0}$, i.e. static field becomes the stationary one.

# EM fields independent on time

## Stationary fields

| | I | II |
|---|---|---|
| 1. | $\text{rot } \mathbf{E} = 0$ | $\text{rot } \mathbf{H} = \mathbf{J}$ |
| 2. | $\text{div } \mathbf{D} = \rho$ | $\text{div } \mathbf{B} = 0$ |
| 3. | $\mathbf{D} = \varepsilon \mathbf{E}$ | $\mathbf{B} = \mu \mathbf{H}$ |
| 4. | $\mathbf{J} = \sigma \mathbf{E} + \mathbf{J}^{(z)}$ | |

Because $\mathbf{J} \neq 0$ electric phenomena are connected with magnetic phenomena

$$\text{rot } \mathbf{H} = \sigma \mathbf{E} + \mathbf{J}^{(z)} . \tag{20}$$

# EM fields independent on time

## Quasi-stationary fields

| | I | II |
|---|---|---|
| 1. | $\text{rot } \mathbf{E} + \dot{\mathbf{B}} = 0$ | $\text{rot } \mathbf{H} = \mathbf{J}$ |
| 2. | $\operatorname{div} \mathbf{D} = \rho$ | $\operatorname{div} \mathbf{B} = 0$ |
| 3. | $\mathbf{D} = \varepsilon \mathbf{E}$ | $\mathbf{B} = \mu \mathbf{H}$ |
| 4. | $\mathbf{J} = \sigma \mathbf{E} + \mathbf{J}^{(z)}$ | |

Fields are not independent, because the Faraday's electromagnetic induction appears. This case find well applications in electro- and radio- technologies.

# EM fields independent on time

## Quasi-stationary fields

It is assumed that fields change slowly, so the effects connected the finite velocity of EM waves are neglected. The plane wave running along the $x$ - axis with the velocity $c$ can be represented in a form

$$E(x,t) = E_0 \exp\left\{ i\omega t - \frac{i\omega x}{c} \right\} . \qquad (21)$$

We develop it into series with respect to $x$

$$E(x,t) = E_0 \exp\left(1 - \frac{i\omega}{c}x + \ldots\right) \exp(i\omega t) . \qquad (22)$$

# EM fields independent on time

## Quasi-stationary fields

It is seen that limitations resulting from the finite velocity $c$ can be neglected if

$$\frac{\omega}{c}x \ll 1\,. \tag{23}$$

Because $\omega/c = 2\pi/\lambda$, where $\lambda$ – length of wave,i.e.

$$x \ll \lambda\,. \tag{24}$$

The electric current in Poland alternates with frequency 50 Hz, so the corresponding wave length is $6 \cdot 10^3$ km, so the retardation effects can be neglected even for very precision electrotechnical devices sending information over the dimensions of our state.

# Koniec? :-(

The end of the lecture 10